Optical multifunction logic gate based on BSO photorefractive crystal

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ABSTRACT

We discuss an electro-optical device that acts as a multifunction logical gate based on a BSO photorefractive crystal. It is an easily re-configurable device which can perform different logic functions such as AND, OR, NOT, NOR using the same configuration and changing only the controlling parameters.

Keywords: Logical device, electro-optical device, logical gate, optical computing device.

1. INTRODUCTION

Lights is very useful to perform high speed all-optical operations\textsuperscript{1}. Different techniques and systems have been proposed but the resulting devices are generally mono-function, that is they are capable of performing only one operation that can be logical, arithmetical, or other.

In this paper we show how it is possible to use a BSO photorefractive crystal to design a multifunction logical gate, that is a re-configurable device which can perform different logic functions such as AND, OR, NOT, NOR using the same configuration and changing only the controlling parameters.

BSO crystals show the interesting property of supporting soliton-like propagation at really low light intensities (few mW). Main feature of this material is the strong optical activity, responsible for energy exchange between polarisations during propagation\textsuperscript{2}. It has been demonstrated\textsuperscript{2,3} that a nonlinear regime can be reached for which this polarisation rotation can be externally controlled. This occurs because to obtain solitons the BSO crystal must be uniformly enlighten with a background beam $I_b$ and biased by a static transversal electrical field $E_0$. The solitonic beam $I_s$ entering the crystal experiences a nonlinear rotation of its polarisation, in the sense that its rotation velocity can be controlled and modified by the background intensity $I_b$, by the electrical field $E_0$, and by its own intensity too. The operating geometry is shown in fig.1.

![Fig.1 Considered disposition of the components.](image)

With a BSO crystal whose length is well defined, it is possible to control the intensity of the two output polarisations acting on the background beam, on the electrical field and on the intensity of the input beam, obtaining full signal condition (logic one) or absence of signal (logical zero) as a function of the intensity resulting by the superposition of two ore more input beams that act on the main beam or on the background beam.
The operation of the device is discussed in the following and it is studied from the numerical point of view showing its peculiar features of multifunction logic gate.

2. PROPERTIES OF BSO CRYSTAL

Anisotropic crystals show two autosolutions for the propagation of two different polarisations each of them characterised by a refractive index (birefringence)\(^4\).

BSO (\(B_2O_3SiO\)) is a crystal characterised by a high optical activity, that makes it able to rotate the direction of polarisation of an electromagnetic wave that propagates in it according to certain directions. It shows a circular anisotropy\(^5\).\(^9\).

The rotation power \(\rho\) of BSO is equal to 38.6°/mm at the wavelength of 514.4 nm.

Let us consider the propagation of a laser beam whose intensity is \(I(x,z)\) in a BSO crystal that is subjected to a transverse constant electrical field \(E_0\) generated through an external voltage \(V\) and that is uniformly enlighten by means of a background beam whose intensity is equal to \(I_{bo}\) and whose propagation direction is inclined with respect to the main beam to avoid to be revealed by the light detector, as shown in fig.1.

The crystal is cut in such a way that the main beam propagates along the \([-1\ 0\ 0]\) (z axes) crystallographic direction and it is free of diffracting along x axes \([0\ 0\ -1]\). The external electrical field \(E_0\) is applied along the same direction.

To derive the equations of propagation it is necessary to start from the Helmholtz equation that describes the propagation of the electric field \(E\) of the wave in the material:

\[
\nabla^2 \tilde{E} + \frac{k_e^2}{\varepsilon_0} \tilde{D} = 0
\]

where \(k_e = \frac{2\pi}{\lambda_0}\) is the wavevector in the vacuum and \(\lambda_0\) the wavelength used. The vector \(\tilde{D}\) is related to the electrical field \(\tilde{E}\) from the following dispersion relation, that is valid in the presence of slowly varying amplitude and in a lossless material:

\[
\tilde{D} = \varepsilon_0 \tilde{E} + \varepsilon_0 \Delta \tilde{E} \tilde{E} + i\varepsilon_0 k_e \tilde{g} \times \tilde{E} .
\]

where \(\Delta \tilde{E}\) represents the variation of the dielectric tensor due to the Pockels effect and \(\tilde{g}\) is the rotation vector of the BSO. If \(E_0\) is the ratio between the external potential \(V\) and the transversal dimension \(W\) in the x direction, the space charge field generated by the diffusion of the carrier in the less enlighten zones in stationary conditions can be approximated as:

\[
\tilde{E}_{sc} = E_{sc} \hat{x} = E_0 \frac{1}{1 + \frac{I(x,z)}{I_{vo}}}
\]

The space charge field produces, through the Pockels effect, a variation of dielectric constant:

\[
(\Delta \tilde{E})_0 = -n_0 \Delta \eta_0
\]

where \(\Delta \eta_0 = r_0 E_{sc}^0\), being \(r_0\) the electrooptical coefficient of the material. In the considered geometry, eq.(4) becomes:

\[
\Delta \tilde{E} = n_0 r_0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -E^{sc} & 0 \\ 0 & 0 & E^{sc} \end{bmatrix}
\]

Using eq.(5), equation (2) can be written as:

\[
\tilde{D} = \varepsilon_0 \tilde{E} + \varepsilon_0 \Delta \tilde{E} \tilde{E} + i\varepsilon_0 \tilde{g} \times \tilde{E}
\]

where \(\tilde{g} = \frac{\rho \lambda_0 n_0}{\pi}\) is a vector directed along the propagation direction of the beam.

If we look for a solution of the Helmholtz equation of the kind:

\[
\tilde{E}(x, z, t) = E_x \hat{x} + E_y \hat{y} = [A_x(x, z) \hat{x} + A_y(x, z) \hat{y}] \exp(i\omega \cdot t - ik \cdot z)
\]
where \( \tilde{k} = n_{0} \hat{k} \), vector \( \tilde{D} \) can be written as:

\[
\tilde{D} = \epsilon_{0} \left[ \begin{array}{c}
\frac{n_{0}^{2}}{\Gamma} - i\Gamma \\
\frac{n_{0}^{2}}{\Gamma} - n_{0}^{2}r_{0}E_{x}E_{y}(x, z)
\end{array} \right] \left[ \begin{array}{c}
E_{x} \\
E_{y}
\end{array} \right]
\]

Substituting eq.(8) into eq.(1) and using the slowly varying envelope approximation we obtain the equations of propagation for both the \( A_{x}, A_{y} \) components of the field:

\[
\begin{align*}
2i\kappa \frac{\partial A_{x}}{\partial z} + \Gamma \frac{\partial^{2} A_{x}}{\partial x^{2}} - i\Gamma A_{x} &= 0 \\
2i\kappa \frac{\partial A_{y}}{\partial z} + \frac{\partial^{2} A_{y}}{\partial x^{2}} + i\Gamma A_{y} - k_{0}^{2}n_{0}^{2}r_{0}E_{0} \frac{A_{y}}{1 + \frac{I(x, z)}{I_{0}}} &= 0
\end{align*}
\]

(9)

where \( I(x, z) = \frac{n_{0}}{2\eta_{0}} \left( |A_{x}|^{2} + |A_{y}|^{2} \right) \) and \( \eta_{0} = \frac{\mu_{0}}{\epsilon_{0}} \).

Eqs.(9) are two coupled equations for the amplitudes \( A_{x}, A_{y} \).

Due to the presence of terms depending on the optical activity of the crystal the velocity of rotation of the polarisations strictly depend on the external electrical field \( E_{x} \) and on the ratio between the intensity of the beam \( I(x,y) \) and the intensity of the background beam \( I_{0} \).

3. NUMERICAL SIMULATIONS

The equations of propagation (9) have been numerically solved using a FD-BMP algorithm.

In figs.2 the numerical simulations of \( x \) polarisation for different values of the external field are shown. The spatial pulsations during the propagation are due to the energy exchange between \( x \) and \( y \) polarisations: it is possible to see that the velocity of rotation of the polarisation increases with the increase of the external electrical field, that is the higher the external field the shorter the oscillation period is with propagation. The propagation length has been considered to be equal to 8 mm while the peak intensity of the beam is equal to 5.8 W/cm².

Different numerical simulations were made to obtain the output intensity of \( x \) polarisations as a function of the ratio between the maximum intensity \( I_{0} \) of the beam \( I(x,y) \) (that is considered to be gaussian) and the intensity of the background beam \( I_{0} \) for different values of the electrical field \( E_{x} \). Since the ratio between the two intensities can be varied keeping alternatively one of the two intensities constant, both situations have been considered.
Figs. 2 Longitudinal view of the numerical simulations of X polarisation for different values of the external electrical field: the higher the field the shorter the oscillation period of the beam. The propagation length has been considered to be equal to 8 mm while the peak intensity of the beam is equal to 5.8 W/cm².

In fig. 3 it is shown the output intensity of x polarisation as a function of the ratio between the maximum intensity $I_M$ of the beam $I(x,y)$ and the intensity of the background beam $I_0$, for different values of the electrical field $E_0$ keeping $I_0$ constant while the situation where $I_M$ is constant is shown in fig. 4. In both cases we obtain output curves that are characterised by a nonlinear behaviour as a function of the ratio between the two intensities, that can be properly used for our purposes, as we will show later.

Fig. 3 Mean output intensity of X polarisation as a function of the ratio between the maximum intensity $I_M$ of the input beam and the intensity of the background beam $I_0$, for different values of the electrical field $E_0$. The intensity of the background beam $I_0$ is constant and it is equal to 5.8 W/cm².
Fig. 4 Mean output intensity of X polarisation as a function of the ratio between the intensity of the background beam $I_{b0}$ and the maximum intensity $I_M$ of the input beam for different values of the electrical field $E_0$. The intensity of the input beam $I_M$ is constant and it is equal to 5.8 W/cm$^2$.

4. IMPLEMENTATION OF THE LOGICAL GATES

Thank to the great variability of the output intensity of x polarisation as a function of the ratio between the two input intensities, it is possible to use the proposed configuration to realise optical multifunction logical gates that are easily configurable to change their function.

Given the disposition shown in Fig. 1, it is possible to act on $I_M$ and $I_{b0}$ or on the external electrical field to generate proper output values of the x polarisation. The external field is used to modulate the response of the output intensity curve of x polarisation, while both the inputs $I_M$ and $I_{b0}$ can be used, keeping the other one constant, as input of two logic variables A and B that are summed by the intensity point of view. The logic variables can assume two values that are low level=0 or high level='signal', or briefly 'sign.'

The low level correspond to the logical zero and the signal level correspond to the logical one.

5. LOGICAL GATES WITH CONSTANT BACKGROUND

Keeping $I_{b0}$ constant, it is possible to use $I_M$ as input of both the logic variables A and B, using a proper bias beam, to obtain an OR gate that gives as output a high value when one or both the inputs are high and an output equal to a low value when both the inputs are equal to zero, as shown in Table 1. The operative curve is shown in Fig. 5.
6. LOGICAL GATES WITH CONSTANT MAIN BEAM

Keeping $I_M$ constant, it is possible to use $I_{b0}$ as input of both the logic variables A and B, using a proper bias beam, to obtain a NOR gate that gives as output a low value when one or both the inputs are high and an output equal to a high value when both the inputs are equal to zero, as shown in table 2. The operative curves are shown in figs.6.

<table>
<thead>
<tr>
<th>Table 1 Operative behaviour of the OR gate with $I_{b0}$ constant.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input parameters</strong></td>
</tr>
<tr>
<td>$I_M$</td>
</tr>
<tr>
<td>Logical outputs</td>
</tr>
</tbody>
</table>

Fig.5 Operative curve of OR gate with $I_{b0}$ constant. The parameters of configuration of the gate are $E_0 = 40$ kV/cm, bias = 0.1 $I_{b0}$, signal = 1.15 $I_{b0}$.

Table 2 Operative behaviour of the NOR gate with $I_M$ constant.

<table>
<thead>
<tr>
<th>Table 2 Operative behaviour of the NOR gate with $I_M$ constant.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input parameters</strong></td>
</tr>
<tr>
<td>$I_{b0}$</td>
</tr>
<tr>
<td>Logical outputs</td>
</tr>
</tbody>
</table>
It is also possible to obtain an AND gate that gives as output a high value when both the inputs are high and an output equal to a low value when one or both the inputs are equal to zero, as shown in table 3. The operative curve is shown in fig.7.
### Table 3 Operative behaviour of the AND gate with $I_M$ constant.

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>$A=0$, $B=0$</th>
<th>$A=0$, $B=1$ (Sign.)</th>
<th>$A=1$ (Sign.), $B=0$</th>
<th>$A=1$ (Sign.), $B=1$ (Sign.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{i0}$</td>
<td>$\text{Bias}$</td>
<td>$\text{Bias+Sign.}$</td>
<td>$\text{Bias+Sign.}$</td>
<td>$\text{Bias+2xSign.}$</td>
</tr>
<tr>
<td>Logical outputs</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3: Operative behaviour of the AND gate with $I_M$ constant.

![Operative curve of AND gate with $I_M$ constant](image)

Fig. 7 Operative curve of AND gate with $I_M$ constant. The parameters of configuration of the gate are $E_0 = 27.5$ kV/cm, bias = $0.6 I_M$, signal = $1.25 I_M$.

It is also possible to obtain a NOT gate that gives as output a high value when the input is high and an output equal to a low value when the input is equal to zero, as shown in table 4. The operative curves are shown in figs. 8.

### Table 4 Operative behaviour of the NOT gate with $I_M$ constant.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>$A=0$</th>
<th>$A=1$ (Sign.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{i0}$</td>
<td>$\text{Bias}$</td>
<td>$\text{Bias+Sign.}$</td>
</tr>
<tr>
<td>Logical outputs</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4: Operative behaviour of the NOT gate with $I_M$ constant.

![Operative curve of NOT gate with $I_M$ constant](image)

Fig. 8a Operative curve of NOT gate with $I_M$ constant. The parameters of configuration of the gate are $E_0 = 27.5$ kV/cm, bias = $0.1 I_M$, signal = $0.9 I_M$. 
It is also possible to obtain an OR gate that gives as output a high value when one or both the inputs are high and an output equal to a low value when both the inputs are equal to zero, as shown in table 5. The operative curve is shown in fig. 9.

<table>
<thead>
<tr>
<th>Input parameters</th>
<th>A=0, B=0</th>
<th>A=0, B=1(Sign.)</th>
<th>A=1(Sign.), B=0</th>
<th>A=1(Sign.), B=1(Sign.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{b0}$</td>
<td>Bias</td>
<td>Bias+Sign.</td>
<td>Bias+Sign.</td>
<td>Bias+2xSign.</td>
</tr>
<tr>
<td>Logical outputs</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5 Operative behaviour of the OR gate with $I_M$ constant.
7. TEMPORAL BEHAVIOUR OF THE DEVICE

All the equations derived in this paper are considered in a stationary situation, that is reached in a time interval that is inversely proportional to the intensity of the input beam. Using intensities of the order of a few $\frac{W}{cm^2}$ the time interval is of the order of 50-100 $\mu sec$. Intensities of the order of $3 \times 10^4 \frac{W}{cm^2}$ give time intervals of the order of 10 $\mu sec$, that are competitive with the electronic velocities.

Since the carrier profile generated by the writing light beam tends to survive for a certain time after that the beam is switched off, it is necessary to delete it using a uniform beam that enlightens the crystal for a time that is almost equal to the time of the writing beam. Therefore, for this kind of application, it is necessary to use a quite high intensity level, to shorten the switching time, and an erasing beam to clean the crystal from the carriers.

8. CONCLUSIONS

We presented and designed an electro optical logical gate, based on the properties of propagation of light beams in BSO crystals.

The device is easily configurable to perform different logical functions such as AND, OR, NOT, NOR without variations of the operative time. The operative velocity is limited by the response time of the photorefractive material and it can be properly increased using appropriate intensity level of the beams.

REFERENCES