ELSEVIER

1 July 1997

Optics Communications 139 (1997) 193–198

OPTICS COMMUNICATIONS

Swing effect of spatial soliton

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Received 6 June 1996; revised 5 November 1996; accepted 6 March 1997

Abstract

A novel transverse oscillatory behavior of a spatial soliton is presented. The controlling parameter of the spatial oscillation period is constituted by the intensity of the input beam. The scheme is based on an interesting property of a soliton in a waveguide with transverse Gaussian refractive index profile.

1. Introduction

Optical solitons mostly arise as self-trapped transverse spatial profiles in nonlinear planar waveguides or as temporal pulses in fibers [1–6]. The interesting property of these propagating wave packets is not only their space-invariant or time-invariant profile, but also their robustness to external perturbations. Spatial and temporal solitons have been observed in different experiments, opening the way to realize, thanks to their interesting properties [7], all-optical devices as arithmetic units [8,9], logical operation [12,13], switching [14–17] and modulation [7,18–21]. In this paper we want to study the propagation of a soliton beam in a waveguide with a transverse Gaussian refractive index profile, with the initial position of the maximum intensity of the soliton shifted with respect to the position of the maximum of the index profile. In this situation the beam is attracted towards the center of the index profile, acquiring a certain velocity that allows it to pass this point and to continue to move forward to the other side of the index profile, decreasing its velocity.

2. Soliton in a Gaussian refractive index profile

Soliton propagation through nonlinear interfaces, that is under the condition of strong perturbation, has already been studied [10,11]. The situation is very similar to the trajectory of a mass in a gravitational potential hole.

It is possible to show that in a plane wave geometry, where the transverse field confinement is given by the refractive index variation, and the refractive index variation between the waveguide and the surrounding media is larger with respect to the refractive index change induced by the nonlinearity, the e.m. propagation is described by the following nonlinear Schrödinger equation (NLSE) in the X–Z plane:

$$2\mathrm{i}\beta\frac{\partial A}{\partial Z}+\frac{\partial^2 A}{\partial X^2}+2\beta^2\frac{n_2}{n_0}|A|^2A=0, \tag{1}$$

Z being the longitudinal propagation coordinate, X the transversal coordinate, A the amplitude of the field and β the wavevector of the guided mode. If a transverse index profile $\Delta n_0(X)$ is present, Eq. (1) changes into:

$$2\mathrm{i}\beta\frac{\partial A}{\partial Z}+\frac{\partial^2 A}{\partial X^2}+2\beta^2\frac{n_2}{n_0(1+\Delta n_0(X))}|A|^2A=0, \tag{2}$$

provided that $|\Delta n_0(X)| \ll 1$, so that the index profile can be regarded as a light perturbation of the NLSE.

It is convenient, for simplicity of calculation, to normalize Eq. (1),

$$2\mathrm{i}\frac{\partial Q}{\partial z}+\frac{\partial^2 Q}{\partial x^2}+\frac{2}{1+\Delta n_0(x)}|Q|^2Q=0, \tag{3}$$

where $\beta X=x$, $\beta Z=z$, $\sqrt{n_2/n_0}A=Q$.

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PII S0030-4018(97)00128-4

We assume that the refractive index profile $\Delta n_0(x)$ is of the form $\Delta n_0 \exp(-bx^2)$, where b is a constant that controls the width of the profile. The index profile has been chosen as Gaussian just for simplicity. In fact the beam oscillations take place even for different profiles because of the particular behavior of the phenomenon we are studying. Further, this kind of profile approximates the index profile obtained in ion exchange waveguides.

Eq. (3) can be rearranged as

$$2\mathrm{i}\frac{\partial Q}{\partial z}+\frac{\partial^2 Q}{\partial x^2}+2|Q|^2 Q=VQ, \tag{4}$$

where V is the perturbation potential that is given by

$$V=2\frac{\Delta n_0\exp(-bx^2)}{1+\Delta n_0\exp(-bx^2)}|Q|^2. \tag{5}$$

The perturbation potential is responsible for the transversal trajectory of the soliton beam. It depends on the index profile and on the intensity profile of the beam: this means that different intensity profiles in the same index profile are subject to different forces and therefore they move according to different trajectories.

We assume that the electromagnetic field moves collectively: that is its local intensity QQ^* is a function only of $x-\bar{x}(z)$. We can thus use the equivalent-particle description as in Ref. [10].

We recall some important general parameters: the exact expression for the rate of change of the normalized dimensionless power:

$$p=\int_{-\infty}^{\infty} QQ^*\,\mathrm{d}x, \tag{6}$$

the average position:

$$\bar{x}=p^{-1}\int_{-\infty}^{\infty} xQQ^*\,\mathrm{d}x, \tag{7}$$

the average velocity:

$$v=\mathrm{i}p^{-1}\int_{-\infty}^{\infty}\left(Q\frac{\partial Q^*}{\partial x}-Q^*\frac{\partial Q}{\partial x}\right)\mathrm{d}x \tag{8}$$

of the normalized field $Q(x,z)$.

In our case it is possible to demonstrate that

$$\mathrm{d}p/\mathrm{d}z=0, \tag{9a}$$

$$\mathrm{d}\bar{x}/\mathrm{d}z=v, \tag{9b}$$

$$a=\mathrm{d}v/\mathrm{d}z=-2p^{-1}\int_{-\infty}^{\infty}\frac{\partial V}{\partial x}QQ^*\,\mathrm{d}x. \tag{9c}$$

Expressions (9a)–(9c) are exact expressions.

It is evident that if we are able to find the analytical expression of Eq. (9c), we can completely describe the behavior of a soliton beam in a refractive index profile.

In our case we suppose that the input wave-packet is a single soliton:

$$Q(x,z)=C\,\mathrm{sech}[C(x-\bar{x})]\exp[\mathrm{i}(vx/2+2\sigma)], \tag{10}$$

where $v=\mathrm{d}\bar{x}/\mathrm{d}z$ and $-v^2/8+\frac{1}{2}C^2=\mathrm{d}\sigma/\mathrm{d}z$.

In this case the potential V expressed by Eq. (5) becomes

$$V=2\frac{\Delta n_0\exp(-bx^2)}{1+\Delta n_0\exp(-bx^2)}\left[C\,\mathrm{sech}(C(x-\bar{x}))\right]^2. \tag{11}$$

Substituting Eq. (11) in Eq. (9c) it is possible to obtain the exact expression of transversal acceleration. Because of the quite complex structure of the potential represented by Eq. (11), it is very difficult to solve in a closed form, the integral (9c), but it is possible, thanks to some approximations, to calculate the analytical expression of the acceleration.

Since $\Delta n_0 \ll 1$, it is possible to neglect it in the denominator of Eq. (5) that can be approximated by

$$V(x)\cong 2\Delta n_0\exp(-bx^2)\left[C\,\mathrm{sech}(C(x-\bar{x}))\right]^2. \tag{12}$$

We restrict our analysis to the situation of a wide index profile with respect to the beam width, that means $b \ll C$. In this situation it is possible to expand the exponential term of Eq. (12) in power series about the point $x=\bar{x}$ to the first order, obtaining

$$V(x)=\left[2\Delta n_0\exp(-b\bar{x}^2)-4b\Delta n_0 x_0\exp(-b\bar{x}^2)\right.$$
$$\left.\times(x-\bar{x})\right]\left[C\,\mathrm{sech}(C(x-\bar{x}))\right]^2. \tag{13}$$

Substituting Eq. (13) into Eq. (9c) we have

$$\begin{aligned}a(\bar{x})=-2p^{-1}\Bigg(&8b\Delta n_0 C^3\bar{x}\exp(-b\bar{x}^2)\\&\times\int_{-\infty}^{\infty}C[\mathrm{sech}(C(x-\bar{x}))]^4\,\mathrm{d}x\\&-4\Delta n_0 C^3\exp(-b\bar{x}^2)\\&\times\int_{-\infty}^{\infty}C[\mathrm{sech}(C(x-\bar{x}))]^4\tanh(C(x-\bar{x}))\,\mathrm{d}x\\&-8b\Delta n_0 C^3\bar{x}\exp(-b\bar{x}^2)\\&\times\int_{-\infty}^{\infty}C(x-\bar{x})[\mathrm{sech}(C(x-\bar{x}))]^4\\&\times\tanh(C(x-\bar{x}))\,\mathrm{d}x\Bigg)\end{aligned} \tag{14}$$

calculating the integral and inserting the minus sign into

Δn_0 we have the acceleration as a function of the variable $\bar{x}$:

$$a(\bar{x}) = -\tfrac{8}{3} b \Delta n_0 C^2 \bar{x} \exp(-b\bar{x}^2), \tag{15}$$

which is always valid under the condition $\Delta n_0 \ll 1$ and $b \ll C$.

We can see that this acceleration is a linear function of Δn_0, a square function of C, an antisymmetric function of $\bar{x}$. In Fig. 1 the optical potential (curve 1), the soliton beam profile (curve 2) when it is positioned at the center of the transverse index profile, and the transversal acceleration (curve 3) are shown.

The parameter b is both responsible for the amplitude and the position $\bar{x}_M$ of the maximum of the acceleration that can be found solving the following equation:

$$\partial a / \partial \bar{x} = 0, \tag{16}$$

whose solutions are

$$\bar{x}_M = \pm 1/\sqrt{2b}\,. \tag{17}$$

In Fig. 2, different transversal acceleration profiles as a function of parameter b are shown.

It is immediate to see that the transverse acceleration (depending on the average position $\bar{x}$) tends to zero when the beam moves out of the waveguide.

If the beam is shifted with respect to the center of the profile and Δn_0 has the proper sign, a force is present that constricts the beam to move towards the center, where it reaches the maximum velocity, than the force inverts its sign when the beam passes through it. As a result we have an oscillatory behavior.

In this paper we are mainly interested in finding the distance that a soliton beam has to propagate along the z-axis to move from one side of the waveguide with an initial velocity equal to zero and to reach the other side of the waveguide with a final velocity equal to zero. Since the

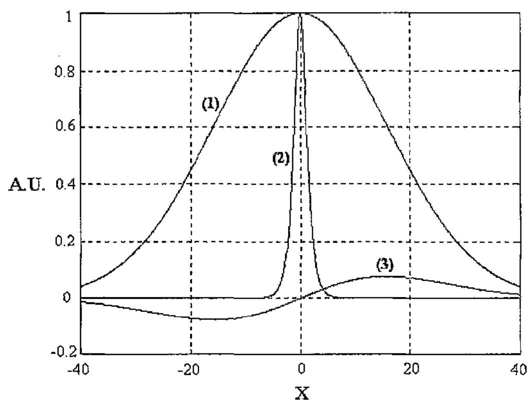

Fig. 1. Optical potential profile (1), soliton beam profile (2), acceleration profile (3) versus transverse coordinate for $b = 0.005$, $C = 1$, $\Delta n_0 = 1$.

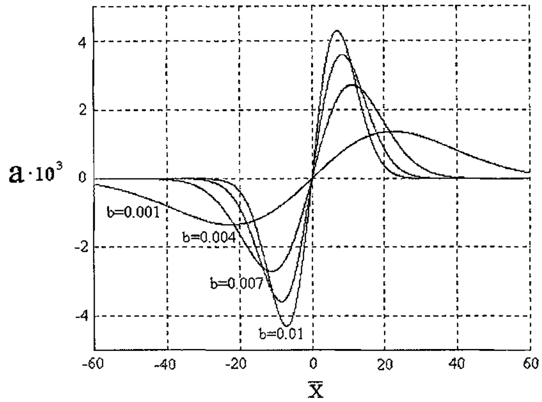

Fig. 2. Acceleration profile versus transverse coordinate for different values of b and $C = 1$, $\Delta n_0 = 0.1$.

transversal acceleration is a function of the variable x, it is convenient to consider the mean acceleration a_M:

$$a_M = \frac{1}{w} \int_0^w a(\bar{x})\, \mathrm{d}\bar{x}, \tag{18}$$

where w is the initial position of the beam with respect to the center of the index profile. In this situation the equation that expresses the transverse distance x as a function of propagation coordinate z is

$$\bar{x} = \tfrac{1}{2} a_M z^2. \tag{19}$$

Since we are interested in knowing z as a function of the transverse path at $\bar{x} = w$, we have from Eq. (19):

$$\tfrac{1}{2} a_M z^2 - w = 0, \tag{20}$$

that can be solved, excluding the non-physical solution, giving:

$$z = S_z/2 = (2w/a_M)^{1/2}, \tag{21}$$

where S_z is the distance necessary to propagate to cross a transverse distance equal to $2w$.

Eq. (19) allows us to exactly know the point of the waveguide where the beam arrives with a velocity equal to zero, before inverting its trajectory. If we consider, for example, the beam to be initially positioned at the maximum of transverse acceleration from Eq. (17) we have that $w = x_M = 1/\sqrt{2b}$. Substituting Eq. (15) in Eq. (18) and solving the integral it is possible to find the analytical expression of a_M:

$$a_M = \frac{1}{1/\sqrt{2b}} \int_{-1/\sqrt{2b}}^{0} \left[-\tfrac{8}{3} b \Delta n_0 \bar{x} \exp(-b\bar{x}^2)\right] \mathrm{d}\bar{x}$$
$$= \tfrac{4}{3}\sqrt{2b}\, \Delta n_0 C^2 \left(1 - \exp(-\tfrac{1}{2})\right). \tag{22}$$

3. Numerical simulations of the effect

We have simulated [11] Eq. (1) for different values of the parameters Δn_0, C, b.

The initial profile was

$$Q_0(x) = C\,\mathrm{sech}[C(x - x_M)], \tag{23}$$

where x_M is the shift between the center of the index profile and the center of the beam, that in our case coincided with the position of the maximum of the transverse acceleration.

The result is that the beam oscillates inside the index profile without leaving it if it is initially properly positioned on one side of the index profile with a transversal velocity equal to zero. For this reason we call this behavior "swing effect".

This behavior perfectly agrees with the theory.

In Fig. 3, different upper views of the results of the simulation for different values of the parameters are shown.

The oscillation period $T_z(C)$ is a function of the parameter C as it was to be expected substituting Eq. (22) into Eq. (21):

$$T_z(C) = \left(\frac{12}{b\Delta n_0\left(1 - \exp\left(-\frac{1}{2}\right)\right)}\right)^{1/2} C^{-1}. \tag{24}$$

4. Numerical example

We want now to consider a numerical situation to give an idea of the strength of the swing effect in a Kerr medium.

We consider a beam whose transverse dimension is d_0, where d_0 is the full width at half height of the beam. In this situation the intensity necessary to generate a soliton is [5]

$$I_s = 2n_0/d_0^2 n_2 \beta^2, \tag{25}$$

where β is the wavevector of the beam, and n_0,n_2 are the linear and nonlinear refractive indices of the medium respectively.

It is possible to demonstrate, through some algebra, that in a profile $C\,\mathrm{sech}(Cx)$ the parameters d_0 and C are linked by the relation:

$$\beta d_0 = \frac{2}{C}\log\left(2 + \sqrt{3}\right). \tag{26}$$

Eq. (26) can be rapidly calculated solving the equation $C\,\mathrm{sech}(Cx) = C/2$, with respect to x, that gives the x-position where the beam is equal to half of its maximum. Excluding the non-physical solution, we have the half of the normalized full width at half height of the beam. Remembering that $\beta X = x$, we obtain Eq. (26).

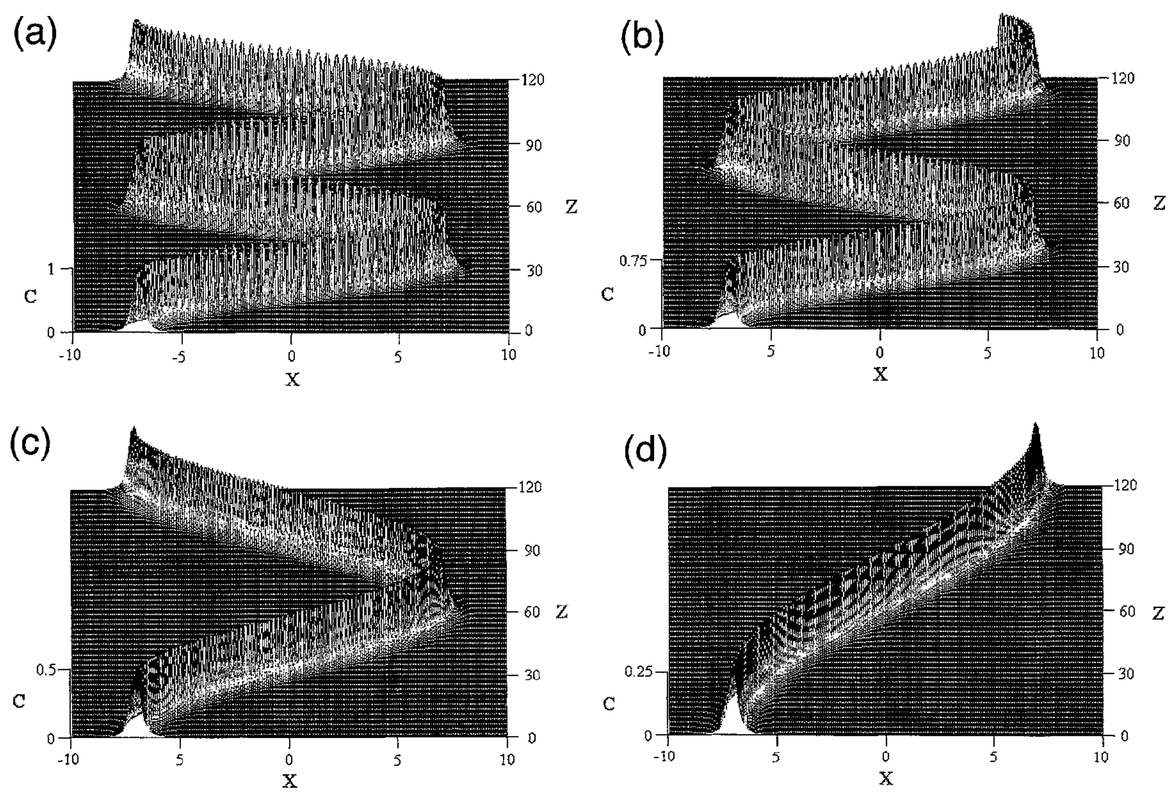

Fig. 3. (a) Numerical simulation for $C = 1$ and $b = 0.01$, $\Delta n_0 = 0.05$. (b) Numerical simulation for $C = 0.75$ and $b = 0.01$, $\Delta n_0 = 0.05$. (c) Numerical simulation for $C = 0.5$ and $b = 0.01$, $\Delta n_0 = 0.05$. (d) Numerical simulation for $C = 0.25$ and $b = 0.01$, $\Delta n_0 = 0.05$.

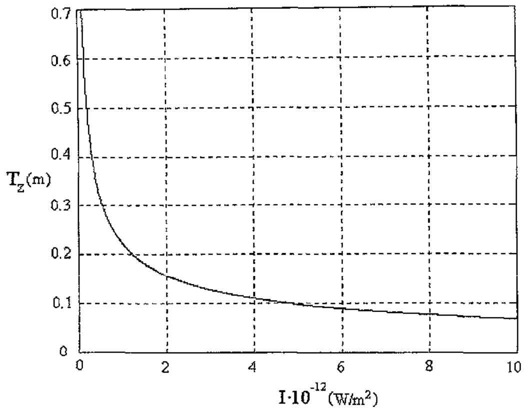

Fig. 4. Spatial oscillation period (meter) as a function of the input intensity (W/m^2).

Substituting Eq. (26) into Eq. (25) and solving with respect to C, we obtain:

$$C = \left(\log(2+\sqrt{3})\right)\left(\frac{2n_2}{n_0}\right)^{1/2} I_s^{1/2}. \tag{27}$$

It is now necessary to transform the condition $b \ll C$, that ensures that the index profile is wider than the beam profile, from normalized unities to real unities, that is to relate b to d_0. The condition holds for example for $b = C/10$. Substituting this last condition into Eq. (26) we have

$$b = \frac{\log(2+\sqrt{3})}{5\beta}\,\frac{1}{d_0}. \tag{28}$$

Once fixed this condition, it is possible to know immediately the width of the index profile W. In fact, if we consider as half of the index profile the transversal distance that extends form the center of the profile until to the point where the index value is equal to $1/e$ of the maximum value, we have that $W = \beta(2/b) = 10d_0/[\log(2+\sqrt{3})]$.

Since the oscillation period is a function of C as expressed in Eq. (24), substituting Eq. (27) and Eq. (28) into Eq. (24), and remembering that $\beta T_Z = T_z$, we have

$$T_Z(I_s) = \left(\frac{60d_0}{\beta\Delta n_0\left[\log(2+\sqrt{3})\right]^3\left[1-\exp(-\frac{1}{2})\right]}\right)^{1/2} \times\left(\frac{n_0}{2n_2}\right)^{1/2} I_s^{-1/2}, \tag{29}$$

that is the spatial oscillation period as a function of the input intensity I_s. It is immediate to verify that Eq. (29) is expressed in meters.

Suppose we have a Kerr material such as a Schott B270 glass [5], whose typical optical parameters at $\lambda_0 = 620$ nm are $n_0 = 1.53$ and $n_2 = 3.4 \times 10^{-20}$ m^2/W, and a spot size whose dimension is $d_0 = 5$ μm. In this situation the width of the index profile W must be chosen equal to about 40 μm. If we choose, for example, $\Delta n_0 = 10^{-2}$, we can calculate the expression that gives the oscillation period as a function of intensity. Substituting the numerical values in Eq. (29) we have

$$T_Z(I_s) = 2.21 \times 10^5 I_s^{-1/2}. \tag{30}$$

In Eq. (30), I_s is expressed in W/m^2 and T_Z in meter. A graphical representation of Eq. (30) is shown in Fig. 4.

5. Conclusions

A new interesting effect involving soliton propagation was presented. It consists of oscillatory behavior of a soliton when it is placed in a transverse Gaussian refractive index profile. The spatial oscillation was demonstrated to be a function of the intensity of the beam. Different index profiles, not considered here, induce oscillations, even if the spatial periods depend on their shape.

Acknowledgements

This work was performed under a 40% MURST Grant.

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