Interference Suppression in MIMO Systems for Throughput Enhancement and Error Reduction

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ABSTRACT

This contribution analyzes the problem of multi-user interference suppression via estimation and subtraction. In particular it is shown that spatial diversity helps the suppression capability of a Base Station and this has an important impact on performance. By fact, we show that the best solution does not consist in transmit at maximum energy level but, in order to allow interference estimation and cancellation, a trade off has to be solved. So, interference suppression can give arise to performance improvements both for achieved throughput (that can be "enhanced") and for error probability (that can be reduced).

Categories and Subject Descriptors

H.1 [**Models and Principles**]: Systems and Information Theory

General Terms

Algorithms, Theory.

Keywords

Multiple Antennas, Multi User Interference, Space Division Multiple Access, Power-Allocation.

1. INTRODUCTION AND SYSTEM MODEL

Multiple-antennas allow to achieve very high Bit Rates and low Bit Error Rate without heavy energy consumption, so in the last years, Multiple-Input Multiple-Output (MIMO) systems appeared to be in a "good standing" in order to be proposed as technology for next generation WLANs and Cellular Networks. Furthermore, the performance gain offered by multiple antennas deals with the multi-user interference (MUI) suppression capability and, linked to this last, multiple-antennas allow the system to exploit spatial diversity in order to resort to new paradigms such as Space Division Multiple Access (SDMA) [9,13,14,16]. In [4], [6] it is proved that the SDMA is able to outperform conventional orthogonal (e.g., interference free) access schemes in several multi-user interfered channels. In this contribution we show the effect of multi-antenna aided interference cancellation on network throughput and error phenomena in order to achieve performance close to the interference free scenario, without proceed with signal shaping at the transmitter. The application scenario we consider models emerging wireless networks where a (large) number of transmit-receive nodes simultaneously attempt to communicate over a limited-size cell and then give arise to MUI. The (complex base-band equivalent) point-to-point radio channel linking a transmit node Tx to the corresponding receive station Rx is sketched in Fig.1. Simply stated, it is composed by a transmit unit equipped with $t_0 \geq 1$ antennas communicating to a receive unit equipped with $r \geq 1$ antennas via a MIMO radio channel impaired by both slow-variant flat Rayleigh fading and additive MUI induced by adjacent transmit nodes active over the same hot-spot cell. The path gain h_{ji} from the transmit antenna i to the receive one j may be modelled as a complex zero-mean unit-variance proper complex random variable (r.v.) [5,11,14,16] and, for sufficiently spaced apart antennas, the path gains $\{h_{ji} \in \mathbb{C}^1,\ 1 \leq j \leq r,\ 1 \leq i \leq t\}$ may be considered mutually uncorrelated. Furthermore, for low-mobility applications as those serving users nomadic over hot-spot cells, the path gains $\{h_{ji}\}$ may be also assumed time-invariant over T $\geq$ 1 signalling periods, after which they change to new *statistically independent* values held for another T signalling periods, and so on. The resulting "block-fading" model well captures the main features of several frequency-hopping or packet based interleaved 4G systems, where each transmitted packet is detected independently of any other [4,14]. About the MAI affecting the link of Fig.1, its statistics mainly depend on the network topology [6], and in the application scenario here considered it is reasonable to assume these last constant over (at least) an overall packet [6]. However, since both path gains $\{h_{ji}\}$ and MUI statistics may change from a packet to another, we assume that Tx and Rx in Fig.1 are not aware of them at the beginning of each transmitted packet. Hence, we assume that the coded and modulated streams radiated by the transmit antennas are split into packets composed by

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T ≥ 1 slots, where the first $\mathrm{T}_L \geq 0$ slots are used by the receiver for learning the MAI statistics (see [4]), the second $\mathrm{T}_{tr} \geq 0$ slots are employed for estimating the path gains $\{h_{ji}\}$ of the forward MIMO channel (see [4]), and the last $\mathrm{T}_{pay} \triangleq \mathrm{T} - \mathrm{T}_{tr} - \mathrm{T}_L$ slots convey payload data (see [4]).

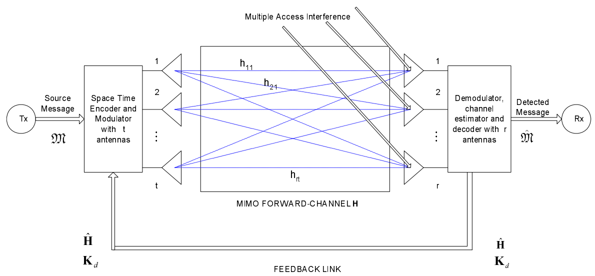

Figure 1: Multi-Antenna system equipped with imperfect (forward) channel estimates $\hat{\mathbf{H}}$ and impaired by MAI with spatial covariance matrix $\mathbf{K}_d = \mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r$.

1.1 The Payload Phase

As in [4] we suppose to get estimation of interference covariance matrix $\mathbf{K}_v$ and channel coefficients matrix $\hat{\mathbf{H}}$ and by basing on these last and actual packet $\mathfrak{M}$ to be transmitted, the transmit node of Fig.1 suitable shapes the signal streams, to be radiated during the payload phase. The corresponding (sampled) signals measured at the outputs of the receive antennas may be modelled as [10,11]

$$\mathbf{Y} = \sqrt{\frac{E_{s0}}{t_0}}\mathbf{\Phi}_0^{(l)}\mathbf{H}_0 + \sum_{n=1}^{U}\sqrt{\frac{E_{Sn}}{t_n}}\mathbf{\Phi}_n^{(l)}\mathbf{H}_n + \mathbf{N} \qquad (1)$$

where the $(T_{pay} \times t_0)$ matrix $\mathbf{\Phi}_0^{(l)}$ is the l-th $(l = 0, ..., M-1)$ transmitted codeword by user 0, $\mathbf{H}_0$ is the $(t_0 \times r)$ matrix collecting the path gains of the link from transmitter 0 and base station, $\mathbf{\Phi}_n^{(l)}$ is the l-th $(l = 0, ..., M-1)$ transmitted codeword by user n, while $\mathbf{H}_n$ is the $(t_n \times r)$ matrix collecting the path gains of the link from transmitter n to base station. Finally $\mathbf{N}$ collects the $(T_{pay} \times r)$ zero-mean $\mathcal{N}_0$-variance complex noise samples modeling Additive White (spatially and temporally) Gaussian Noise (AWGN). The presence of interference presents two worsening effects. The first one is a reduction of the throughput region [6] and the second one consists in to increase the Bit Error Probability (BEP). These drawbacks may be counterbalanced by adopting signal shaping techniques [6] and by trying to suppress interference as shown in the following. So the ultimate goal may be to maximize the *net throughput* (also known as *goodput*) that, according to the ARQ systems, can be rewritten as

$$\mathfrak{R} \triangleq \mathbb{T}(1 - P_E) \ \ (bits/slot), \qquad (2)$$

that is the rate weighted by BEP in order to consider the number of correctly detected bits.

2. MULTI USER INTERFERENCE MIMO SUPPRESSION

By considering the Gaussianity of noise $\mathbf{N}$ we have that, by defining the interference term due to the presence of users as in the following expression (see also [16] and reference therein)

$$\mathbf{V} \triangleq \sum_{n=1}^{U}\sqrt{\frac{E_{Sn}}{t_n}}\mathbf{\Phi}_n^{(l)}\mathbf{H}_n, \qquad (3)$$

we can proceed with optimal (and linear) estimation of interference so to have

$$\tilde{\mathbf{V}} = \mathbf{Y}\mathbf{A}, \qquad (4)$$

where $\mathbf{A}$ is a $(r \times r)$ matrix. In order to derive this last we have to proceed by applying the Orthogonal Projection Lemma and to require that

$$\mathrm{E}\left\{(\tilde{\mathbf{V}} - \mathbf{V})^{\dagger}\mathbf{Y}\right\} = \mathbf{0}_{r\times r}, \qquad (5)$$

that can be rewritten as

$$\mathrm{E}\left\{(\mathbf{Y}\mathbf{A} - \mathbf{V})^{\dagger}\mathbf{Y}\right\} = \mathrm{E}\left\{\mathbf{A}^{\dagger}\mathbf{Y}^{\dagger}\mathbf{Y}\right\} - \mathrm{E}\left\{\mathbf{V}^{\dagger}\mathbf{Y}\right\} = \mathbf{0}_{r\times r}, \qquad (6)$$

that leads to

$$\mathbf{A}^{\dagger}\mathrm{E}\left\{\mathbf{Y}^{\dagger}\mathbf{Y}\right\} = \mathrm{E}\left\{\mathbf{V}^{\dagger}\mathbf{Y}\right\}. \qquad (7)$$

Before to proceed some consideration can be pointed out. The term at l.h.s. of eq.(7) can be rewritten in the following way

$$\mathrm{E}\left\{\mathbf{Y}^{\dagger}\mathbf{Y}\right\} \triangleq \mathrm{E}\left\{\frac{E_{s0}}{t_0}\mathbf{H}_0^{\dagger}\mathbf{\Phi}_0^{\dagger}\mathbf{\Phi}_0\mathbf{H}_0 + \right.$$

$$\left. + \sum_{n=1}^{U}\frac{E_{sn}}{t_n}\mathbf{H}_n^{\dagger}\mathbf{\Phi}_n^{\dagger}\mathbf{\Phi}_n\mathbf{H}_n + \mathbf{N}^{\dagger}\mathbf{N}\right\}, \qquad (8)$$

where, under the hypothesis of orthogonal space-time codes $\mathbf{\Phi}_n^{\dagger}\mathbf{\Phi}_n = \mathbf{I}_{t_n}$ $(n = 0, ..., U)$ we can pose

$$\mathbf{R}_h = \frac{E_{s0}}{t_0}\mathrm{E}\left\{\mathbf{H}_0^{\dagger}\mathbf{H}_0\right\} \qquad (9)$$

and

$$\mathbf{K}_v = \sum_{n=1}^{U}\frac{E_{sn}}{t_n}\mathrm{E}\left\{\mathbf{H}_n^{\dagger}\mathbf{H}_n\right\} \qquad (10)$$

and this leads to the following expression for $\mathbf{A}$

$$\mathbf{A} = \left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_h\right)^{-1}\right)^{\dagger}. \qquad (11)$$

About the performance of this estimator, the estimation error variance $\sigma_{\varepsilon v}^2 = Tra\{\mathrm{E}\{(\tilde{\mathbf{V}} - \mathbf{V})^{\dagger}(\tilde{\mathbf{V}} - \mathbf{V})\}\}$ is given by

$$\sigma_{\varepsilon v}^2 = Tra\left\{\mathrm{E}\{\mathbf{V}^{\dagger}\mathbf{V}\}\right\} + Tra\left\{\mathrm{E}\{\tilde{\mathbf{V}}^{\dagger}\tilde{\mathbf{V}}\}\right\} - 2\,Tra\left\{\mathrm{E}\{\tilde{\mathbf{V}}^{\dagger}\mathbf{V}\}\right\} \qquad (12)$$

that can be finally expressed as

$$\sigma_{\varepsilon v}^2 = Tra\{\mathbf{K}_v\} + Tra\{\mathbf{K}_v(\mathbf{K}_v(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_h)^{-1})^{\dagger}\} -$$

$$-2\,Tra\{(\mathbf{K}_v(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_h)^{-1})^{\dagger}\mathbf{K}_v\} \qquad (13)$$

Remark - About the interference suppressor performances
By observing the expression in eq.(13) it appears clear that when $\mathcal{N}_0 >> 1$ (or $Tra\{\mathbf{R}_h\} >> 1$) we have $\sigma_{\varepsilon v}^2 = Tra\{\mathbf{K}_v\}$, while when $\mathcal{N}_0 << 1$ and $Tra\{\mathbf{R}_h\} << 1$, $\sigma_{\varepsilon v}^2 = 0$. ■
Now by defining the Signal-to-Interference-Ratio as

$$SIR_0 = \frac{E_{s0}}{Tra\{\mathbf{K}_v\} + \mathcal{N}_0} \qquad (14)$$

we can appreciate the the gain offered by interference suppression in eq.(14) so to arrive at

$$SIR_0^{(c)} = \frac{E_{s0}}{\sigma^2_{\varepsilon v} + \mathcal{N}_0}. \tag{15}$$

By observing eqs. (13) and (15), it is possible to appreciate how E_{s0} influences the level of $SIR_0^{(c)}$ since it appears at numerator and at (implicitly) at denominator[1] and this gives arise to a behavior different with respect to standard detection ones.
By observing Fig.2, in a "static" situation, that is when only the user 0 is able to change the power level, we experience, for low values of $P = E_{s0}/T_{pay}$, the same performance of the case when only noise is present because $SIR^{(c)}$ in (15) approaches $E_{s0}/\mathcal{N}_0$ and $\sigma^2_{\varepsilon v} \rightarrow 0$. By considering increasing values of P, the term $\sigma^2_{\varepsilon v}$ is no more close to zero so the performances fall till to the case of no interference suppression. It is important to note that, for very high values of P (and low values of $Tra\{\mathbf{K}_v\}$ not shown in Fig.2) the three curves become indistinguishable. This shows that a power control policy suggests to mit not at the maximum power level.
The above remark underlines that the interference suppression strictly depends on the power transmitted by the reference transmitter. Now, by looking at eq. (2) we want to show how is the influence of $SIR^{(c)}$ on the actual performance. After defining

$$\mathbf{K}_{\varepsilon v} \triangleq \mathrm{E}\{(\tilde{\mathbf{V}} - \mathbf{V})^\dagger(\tilde{\mathbf{V}} - \mathbf{V})\}, \tag{16}$$

we have that $\mathbb{T}$ in eq. (2) can be written as

$$\mathbb{T}(\mathbf{H}_0) = \log\det\left[\mathbf{I}_r + \frac{E_{s0}}{t_0}\mathbf{H}_0^\dagger \mathbf{R}_{\phi_0}\mathbf{H}_0\left(\mathcal{N}_0\mathbf{I}_r + \mathbf{K}_{\varepsilon v}\right)^{-1}\right], \tag{17}$$

where the term $\mathbf{R}_{\phi_0}$ takes care of power allocation over t_0 antennas and reduces itself to identity matrix when orthogonal space-time codes are taken into account.

3. THROUGHPUT ANALYSIS AND SUM RATE

Before to proceed, a consideration about complexity should be carried out. This approach is similar to the well know water-filling one but a distinguish has to be made. The main difference is that, in this case, the noise (thermal noise plus residual interference) is not fixed but depends on allocated power, in fact, after defining

$$\mathbf{R}_{h'} \triangleq \mathrm{E}\left\{\mathbf{H}_0^\dagger\mathbf{H}_0\right\} = \frac{E_{s0}}{t_0}\mathbf{R}_h, \tag{18}$$

we can rewrite eq.(17) as (see also footnote 1, [3,7,8])

$$\mathbb{T}(\mathbf{H}_0) = \log\det\Bigg[\mathbf{I}_r +$$

$$\frac{E_{s0}}{t_0}\mathbf{R}_{h'}\Bigg[\mathcal{N}_0\mathbf{I}_r + \mathbf{K}_v + \mathbf{K}_v\left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \frac{E_{s0}}{t_0}\mathbf{R}_{h'}\right)^{-1}\right)^\dagger$$

$$-2\left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \frac{E_{s0}}{t_0}\mathbf{R}_{h'}\right)^{-1}\right)^\dagger \mathbf{K}_v\Bigg]^{-1}\Bigg]. \tag{19}$$

This means that the optimization problem is scalar, because the only parameter to be set in order to maximize this function is E_{s0}, since we assume that all antennas transmit at the same energy level. If we want to proceed with optimization and in particular by recalling optimal power allocation, we have to consider the following expression in place of eq.(19)

$$\mathbb{T}(\mathbf{H}_0) = \log\det\Bigg[\mathbf{I}_r +$$

$$\mathbf{R}_{\phi_0 h}\Bigg[\mathcal{N}_0\mathbf{I}_r + \mathbf{K}_v + \mathbf{K}_v\left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_{\phi_0 h}\right)^{-1}\right)^\dagger$$

$$-2\left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_{\phi_0 h}\right)^{-1}\right)^\dagger \mathbf{K}_v\Bigg]^{-1}\Bigg], \tag{20}$$

where $\mathbf{R}_{\phi_0 h}$ has not the same elements on its diagonal. This problem is vectorial and for this reason more complex to be solved. About the distance between the optimal matrix $\mathbf{R}_{\phi_0 h}^{(opt)}$ and the solution achieved with the standard water-filling approach $\mathbf{R}_{\phi_0 h}^{(wf)}$ we can affirm that these two matrices are in general different because they experience different level of noise. In particular, when we consider $\mathcal{N}_0 >> 1$ or $E_{s0} >> 1$ then we have $\mathbf{R}_{\phi_0 h}^{(opt)} \approxeq \mathbf{R}_{\phi_0 h}^{(wf)}$ and we have the same[2] when $Tra\{\mathbf{K}_v\} << 1$ while, when we consider medium values of $SIR^{(c)}$ the two matrices are considerably different. Up to now, only a "static" situation is considered

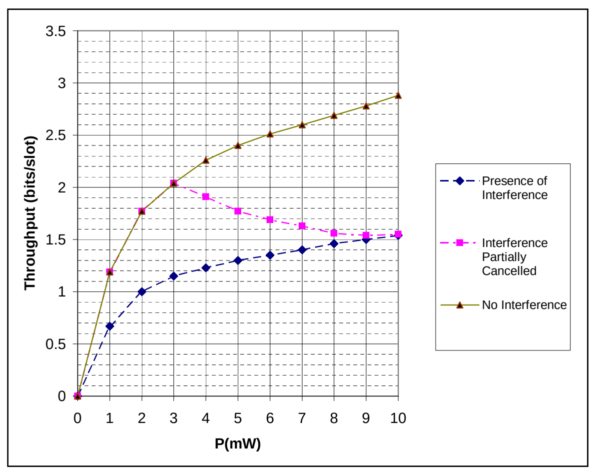

Figure 2: Throughput offered by MIMO system when interference is absent, present and partially cancelled.

and, by considering a centralized control, the optimization should deal with the network throughput, in fact the above

[1]The value assumed by $\sigma^2_{\varepsilon v}$ depends on E_{s0} (see eqs.(9) and (13)). Just as example, if we assume $t_n = r = 1$, $n = 0, ..., U$, we have that $\sigma^2_{\varepsilon v} = k_v - k_v^2(k_v + \mathcal{N}_0 + E_{s0})^{-1}$ and consequently $SIR_0^{(c)} = E_{S0} * (\mathcal{N}_0 + k_v - k_v^2(k_v + \mathcal{N}_0 + E_{s0})^{-1})^{-1}$, where k_v is the interference power.

[2]This directly derives from eq.(13). In fact, when $\mathcal{N}_0 >> 1$ or $E_{s0} >> 1$ no cancellation is possible, so $SIR_0 \approxeq SIR_0^{(c)}$, while, when $Tra\{\mathbf{K}_v\} << 1$ the presence of interference is negligible so $SIR_0 \approxeq SIR_0^{(c)}$.

description refers to the allocation performed by a single user when MUI is present while the main purpose should be the sum rate so to achieve the maximum rate in the network. We can define the function to be maximized as

$$\mathbb{T}^{(tot)}(\mathbf{H}_0, ..., \mathbf{H}_U) = \sum_{n=0}^{U} \mathbb{T}_n(\mathbf{H}_n), \tag{21}$$

that is the summation of the throughput of each link and can be rewritten as

$$\mathbb{T}^{(tot)}_{max}(\mathbf{H}_0, ..., \mathbf{H}_U) = \max_{\substack{E_{sn} \leq E_{sn}^{tot} \\ 0 \leq n \leq U}} \sum_{n=0}^{U} \log\det\Bigg[\mathbf{I}_r + \mathbf{R}_{\phi_n h}\Bigg[\mathcal{N}_0\mathbf{I}_r + \mathbf{K}_v + \mathbf{K}_v\left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_{\phi_n h}\right)^{-1}\right)^{\dagger} -2\left(\mathbf{K}_v\left(\mathbf{K}_v + \mathcal{N}_0\mathbf{I}_r + \mathbf{R}_{\phi_n h}\right)^{-1}\right)^{\dagger}\mathbf{K}_v\Bigg]^{-1}\Bigg] \tag{22}$$

and this can be employed (under the simplifying assumption of $t_n = 1,\ n = 0, ..., U$) also for power control in conventional single antenna terminal applications [1,4]. By considering the simple scenario of two simultaneous active links, in Fig.3 it is possible to appreciate how the interference cancellation and signal shaping, is able to improve performance. In Fig.3 the throughput region [2,6,12] by considering two users, is depicted and the gain achieved by processing is easily recognizable. In fact, the "triangular" region is that taking into account for no interference cancellation ($SIR = 0db$) and no signal shaping while, through cancellation and simultaneous signal shaping, the system is able to offer to users a rate of 4 bit/slot in place of 3 (this means a gain of 33.3% in throughput). If only signal shaping is adopted (without any form of interference suppression) the performance, from the throughput region point of view, are better with respect to the case of no shaping and no suppression but worse when compared to the case of suppression and simultaneous shaping. An interesting additional feature of this problem is linked to the maximization and to the geometrical features of the region. It is important to observe that, by assuming homogeneous network (that is all the users present the same parameters) if the region is "contained" into the quarter of circle, then the max in eq.(22) is represented by the rate allocation $(C_1, 0)$ or $(0, C_2)$ because the each point on the edge of the region is shorter than the above mentioned vectors[3]. On the contrary, when the region "contains" the quarter of circle, then the maximum is the point on the edge that presents the maxiumum gradient (in this particular case of homogeneous network, this means that the maximum is the crossing point between edge and line $R_1 = R_2$). Obviously this approach can be extended to the general case of $U > 2$ and all the above mentioned properties can be founded by substituting the quarter of circle with the orthant of the iper-sphere of U dimension ($\underline{\mathbb{T}} \in \mathbb{R}_+^U$).

[3]Note that $C_n,\ n = 0, ..., U$ is the channel capacity of link n when only noise is present.

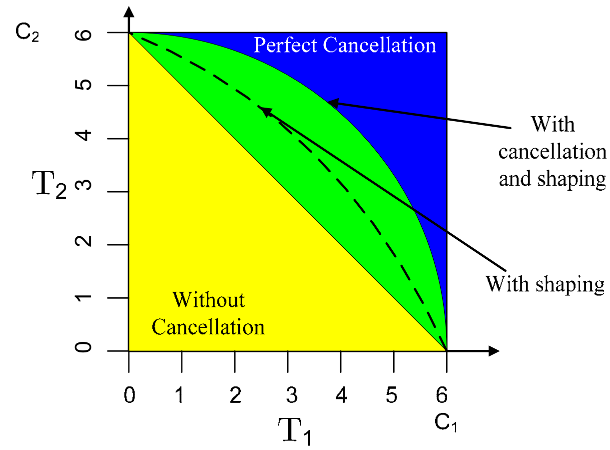

Figure 3: Throughput region improvements with cancellation.

4. ERROR PERFORMANCE

By considering now, the performances form the BEP point of view, we have to consider that the receiver tries to detect the space-time transmitted codeword after interference cancellation so it operates on $\widehat{\mathbf{Y}}$ where it is defined as the $(T_{pay} \times r)$ matrix

$$\widehat{\mathbf{Y}} = \mathbf{Y} - \tilde{\mathbf{V}} = \mathbf{Y}(\mathbf{I}_r - \mathbf{A}) \tag{23}$$

and we suppose the receiver able to perform Maximum Likelihood (ML) detection according to the following rule [5]

$$\hat{\mathbf{\Phi}}_{ML} = \underset{1\leqslant m\leqslant L}{\arg\min}\left\{\mathrm{r}\lg\left[\det\left(\mathbf{Cov}_{\underline{\mathbf{y}}}\left(\mathbf{\Phi}_m\right)\right)\right] + \tilde{z}_m\right\} \tag{24}$$

where the m-th real decision statistic $\tilde{z}_m$ is given by

$$\tilde{z}_m \triangleq \frac{E_{s0}}{t_0} Tra\left\{\left(\mathbf{\Phi}_m\hat{\mathbf{H}}\right)^{\dagger}\left(\mathbf{Cov}_{\underline{\mathbf{y}}}\left(\mathbf{\Phi}_m\right)\right)^{-1}\left(\mathbf{\Phi}_m\hat{\mathbf{H}}\right)\right\} - \frac{\sigma}{t_0} Tra\left\{\left(\mathbf{\Phi}_m^{\dagger}\widehat{\mathbf{Y}}\right)^{\dagger}\left(\mathbf{Cov}\left(\mathbf{\Phi}_m\right)\right)\left(\mathbf{\Phi}_m^{\dagger}\widehat{\mathbf{Y}}\right)\right\} + -2\sqrt{\frac{E_{s0}}{t_0}}\,\mathrm{Re}\left\{Tra\left\{\left(\mathbf{\Phi}_m\hat{\mathbf{H}}\right)^{\dagger}\left(\mathbf{Cov}_{\underline{\mathbf{y}}}\left(\mathbf{\Phi}_m\right)\right)^{-1}\widehat{\mathbf{Y}}\right\}\right\}, \tag{25}$$

$\mathbf{Cov}\left(\mathbf{\Phi}_m\right)$ is the $(t \times t)$ semidefinite-positive Hermitian matrix defined as

$$\mathbf{Cov}\left(\mathbf{\Phi}_m\right) \triangleq \left(\mathbf{I}_{t_0} + \frac{\sigma_\varepsilon^2 E_{s0}}{t_0}\mathbf{\Phi}_m^{\dagger}\mathbf{\Phi}_m\right)^{-1}, 1 \leqslant m \leqslant L \tag{26}$$

and it may compute the inverse matrix $\left(\mathbf{Cov}_{\underline{\mathbf{y}}}\left(\mathbf{\Phi}_m\right)\right)^{-1}$ as

$$\left(\mathbf{Cov}_{\underline{\mathbf{y}}}\left(\mathbf{\Phi}_m\right)\right)^{-1} = \mathbf{I}_{T_{pay}} - \frac{\sigma}{t_0}\mathbf{\Phi}_m\mathbf{Cov}\left(\mathbf{\Phi}_m\right)\mathbf{\Phi}_m^{\dagger}. \tag{27}$$

According to the previous assumption we can evaluate BEP as in [3]

$$P_E = \left(\frac{4t(\mathcal{N}_0 + \sigma_{\varepsilon v}^2)}{1 + E_{s0}2^{-2q}}\right)^r \tag{28}$$

where q is the number of bits/slot employed in the space-time code. In Fig.4 the performances are shown and it is

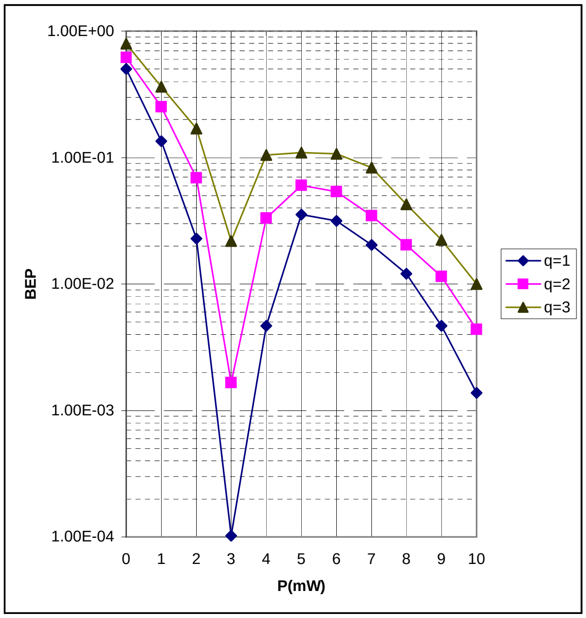

Figure 4: BEP with interference cancellation.

evident the effect of interference cancellation. At low power level, we have the interference is fully cancelled but, at the same time, the level of power is comparable with noise, so the performance, for $P < 1mW$ are influenced by noise. When the emitted power increases, the interference is still cancelled and the power is bigger than noise so we experience a considerable BEP decrease. Finally, when we consider $P < 5mW$ the effect of noise is negligible, while the effect of interference is considerable because we are no more able to estimate and cancel it. Obviously, when we consider high values of P the BEP decreases because noise and interference become negligible (it requires several Watts).

5. NET THROUGHPUT ANALYSIS

As previously anticipated we are interested into evaluate the net throughput already introduced in eq.(2) and known in literature as goodput. Now by considering the throughput and the expression for error probability we have that the goodput for the single link can be expressed by (2)

$$\Re = \log\det\left[\mathbf{I}_r + \frac{E_{s0}}{t_0}\mathbf{H}_0^\dagger \mathbf{R}_{\phi_0}\mathbf{H}_0\left(\mathcal{N}_0\mathbf{I}_r + \mathbf{K}_{\varepsilon v}\right)^{-1}\right]\times$$

$$\times\left[1-\left(\frac{4t_0(\mathcal{N}_0+\sigma_{\varepsilon v}^2)}{1+E_{s0}2^{-2q}}\right)^r\right]. \tag{29}$$

and from this last follows that the parameter q is the throughput $\mathbb{T}$. Now, since the space-time codes have to present codewords gathering symbols of integer bit constellation, this means that the set of power that we can employ at the transmitter is not composed infinite elements by only by the values that give integer values of $\mathbb{T}$, so to maximize the goodput does not imply to find the maximum of eq. (29) but choose the maximum between the power levels allowing integer values of throughput. From a network point of view it appears clear that the main goal should be the maximization of the "sum goodput" that we can defined as the sum of the goodput of each link as

$$\Re_{\sum} = \sum_{n=0}^{U}\log\det\left[\mathbf{I}_r + \frac{E_{sn}}{t_n}\mathbf{H}_n^\dagger \mathbf{R}_{\phi_n}\mathbf{H}_n\left(\mathcal{N}_0\mathbf{I}_r + \mathbf{K}_{\varepsilon v}\right)^{-1}\right]\times$$

$$\times\left[1-\left(\frac{4t_n(\mathcal{N}_0+\sigma_{\varepsilon v}^2)}{1+E_{sn}2^{-2q}}\right)^r\right], \tag{30}$$

where $\mathbf{K}_{\varepsilon v}$ is different for each n. As previously stated, the maximization of eq. (30) does not mean that the throughput (goodput) achieved through a centralized control presents fairness properties because it strictly depends on the interference level, number of users and other parameters (see Sect.III).
By observing the performance plots of Fig.5 it is possible to appreciate that when the number of users in the network increases, the receiver is able to split the signal (this is true if the number of users is less then the number of users). In particular, when we consider (i.e. $r = 8$) can we appreciate that the goodput does not fall since $r > U$ and it considerably reduced itself (it decreases from 23.23 to 20 bits/slot) when $r < U$. When we consider $U >> r$ the becomes Gaussian and spatially white (by invoking the Central Limit Theorem) so the interference estimator no longer is able to estimate and suppress interference so the goodput presents a floor. When we consider lower number of receive antennas at the BS, the knee in the performance plots appears at a lower value of the number of users. This means that by equipping a Base Station with a high number of antennas, without heavy signal processing, we are able to offer, through interference cancellation, the users with a goodput that is higher than that achievable only with with shaping (see Fig.3). Future developments are going to deal with techniques able to take care not only of average throughput but also of fairness features.

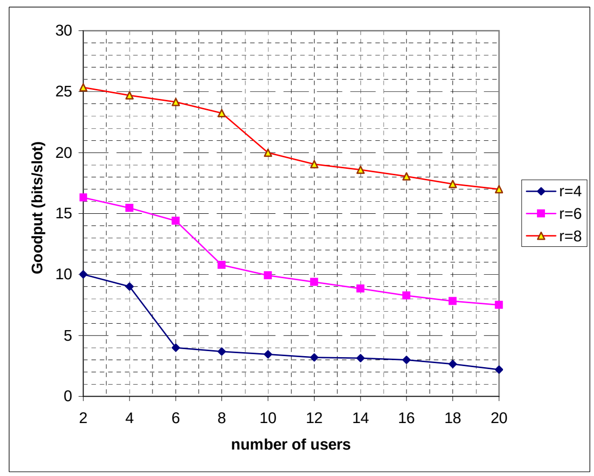

Figure 5: Average goodput for different users and number of receive antennas

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