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High pass intensity controlled soliton filter

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Abstract

A device that allows to pass only a soliton beam characterized by an intensity greater than a certain threshold is presented. The device acts as a high pass filter. It is even a useful demultiplexer since beams with amplitude below threshold are expelled from the device with an angle that depends on their intensity.

1. Introduction

There is a growing interest in optical solitons for applications in all-optical processing. Spatial solitons are self-trapped optical beams that propagate without changing their spatial shape, since the diffraction and the nonlinear refraction balance each other in a self-focusing medium [1].

Recently a new generation of spatial optical switches has been proposed that uses the interaction between two bright or dark soliton beams, and waveguide structures that are induced by these interactions [2–8]. The initial conditions, usually used for such interactions, have the form of two distinct parallel solitons. It is well known that when two distinct bright spatial solitons are launched parallel to each other, the interaction force between them depends on their relative distance and on their relative phase [9,10].

Thanks to the peculiar properties of solitons, it has been possible to design a variety of useful devices. One of the most important features is their particle-like behaviour and their relative robustness to external disturbances. This allows one to address a soliton, in the presence of a transverse refractive index variation, towards a fixed path, since the index variation acts as a perturbation against which the soliton reacts as a particle, moving as a packet without any loss of energy.

In this paper we study the behaviour of a soliton beam in a waveguide which, in the plane between the cladding and the substrate, has a distribution of refractive index that follows a trapezoidal curve (we call it trapezoidal waveguide), considering the possibility that the soliton propagates in an oblique direction.

The theory begins with the study of the transverse behaviour of a soliton in a trapezoidal profile, whose longitudinal axis is parallel to the propagation direction of the beam. Once the expression of the acceleration is derived, we investigate the situation characterized by some angular tilt between the soliton propagation direction and the waveguide axis, which is the goal of this paper.

2. Transverse effect of a soliton beam in a trapezoidal shaped refractive index profile

Interesting effects have been found in the study of transverse effects of soliton propagation at the interface between two nonlinear materials [11–13] or in a material in the presence of a Gaussian refractive index profile, that is in low perturbation regime [14].

Here we focus our attention on a trapezoidal index profile for two main reasons. The first reason is that this kind of profile approximates very well the profile of a channel waveguide produced by an ion-exchange process which is characterized by a constant index in the central zone and a quasi-linear slope index in the lateral sides, due to transverse ion diffusion effects. The second reason is that the soliton, when is positioned in a linear index gradient, is subject to a constant acceleration, as we will

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show, which greatly simplifies the development of the theory.

We consider a transverse refractive index profile $\Delta n_0(x)$ of the kind:

$$\Delta n_0(x) = \begin{cases} 0, & x < -b \\ \frac{\Delta n_0}{b-a}x + \frac{\Delta n_0}{b-a}b, & -b \le x < -a \\ \Delta n_0, & -a \le x \le a \\ -\frac{\Delta n_0}{b-a}x + \frac{\Delta n_0}{b-a}b, & a < x \le b \\ 0, & x > b \end{cases} \quad (1)$$

where $2b$ is the total waveguide width, $2a$ is the width of the central constant index profile region, Δn_0 is the maximum index variation. The profile expressed by Eq. (1) is shown in Fig. 1.

Since we are studying a soliton beam, the expression of the field Q at the beginning of the structure is:

$$Q(x,0) = C\,\mathrm{sech}[C(x-\bar{x})], \quad (2)$$

where $\bar{x}$ is the position of the centre of the beam and C is a real constant from which both the width and the amplitude of the field depend.

Due to the anti-symmetric nature of the structure we derive the theory when the beam is positioned on the left side of the waveguide, being only necessary to change the sign of the obtained acceleration, when the beam is positioned on the other side.

It is possible to demonstrate [15,16] that when the soliton beam is positioned in a ramp potential it is subject to a transverse acceleration that depends on its amplitude C. In our case we have:

$$a_T = \frac{2\Delta n_0}{b-a}C^2. \quad (3)$$

The above expression for the acceleration shows two interesting properties: it does not depend on the position $\bar{x}$ of the beam inside the considered region and it depends on the square of input amplitude C. This last property implies that different amplitude beams are subject to different transverse accelerations.

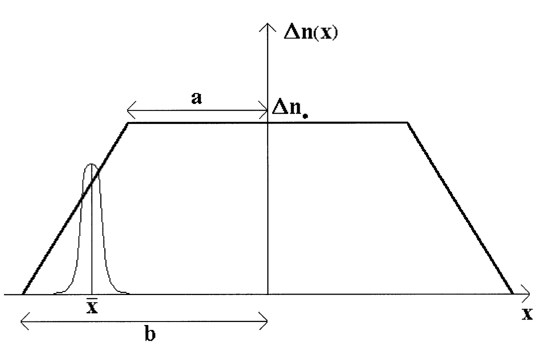

Fig. 1. Section of the ideal trapezoidal refractive index profile waveguide.

We also suppose that the extension of the lateral side of the waveguide is wider than the soliton width, to reduce to a minimum the effect of the discontinuity of acceleration due to the refractive index variation at the beginning and at the end of the lateral zone. This is equivalent to say that $b \gg C$.

3. Transverse effect in an oblique waveguide

We now extend the results obtained in the previous section to the case of a longitudinal inclined waveguide with respect to the soliton propagation direction.

We consider, without any loss of generality, a waveguide characterized by only two lateral sides, without the central part characterized by a constant refractive index and therefore by an acceleration equal to zero. In this way the acceleration is always different from zero inside the waveguide.

It is very useful to use a dynamic point of view to study this situation, that is to consider the step by step transverse relative position of the waveguide with respect to the beam, using the z variable as a time parameter. Under these conditions, the inclination angle α of the waveguide with respect to the longitudinal axis z can be regarded as the transverse velocity of the waveguide:

$$v_G = \frac{\mathrm{d}x_G(z)}{\mathrm{d}z} = \tan\alpha, \quad (4)$$

where $x_G(z)$ is the position of the central part of the waveguide profile with respect to z. From Eq. (4) it is possible to derive x_G as

$$x_G = \int_0^z v_G\,\mathrm{d}\zeta = v_G z, \quad (5)$$

$x_G(z)$ is the position of the centre of the waveguide as a function of z.

The same procedure can be applied for the transverse velocity and the position of the beam using Eq. (3):

$$v_B = \int_0^z a_T\,\mathrm{d}\zeta = \frac{2\Delta n_0}{b}C^2 z, \quad (6a)$$

$$x_B = \int_0^z v_B\,\mathrm{d}\zeta = \frac{\Delta n_0}{b}C^2 z^2. \quad (6b)$$

Let us consider the initial condition of the beam positioned in the centre of the waveguide. At the beginning of propagation, since the waveguide moves with a constant velocity v_G, the soliton beam enters in the constant acceleration zone, where it acquires a velocity linearly increasing with z, according to Eq. (6a). This process continues until the beam remains in this part of the waveguide.

After a certain propagation distance two different situations may occur: the beam leaves the acceleration zone without reaching the velocity v_G of the waveguide or the beam acquires a velocity that is greater than or equal to the velocity v_G of the waveguide. We call the first event ‘detach situation’, since the beam leaves the waveguide with a certain velocity, and therefore a certain angle, that is smaller than the angle of the waveguide. We call the second event ‘lock-in situation’ since the beam reaches the other side of the waveguide where it is stopped, reversing again the situation and so on, but it remains locked inside the waveguide. Considering the situation at general z, as shown in Fig. 2, if we calculate the distance d_{GB} between the waveguide centre and the beam:

$$d_{GB} = x_G - x_B = v_G z - \frac{\Delta n_0 C^2}{b} z^2. \tag{7}$$

The detach condition is obtained when

$$d_{BG} = b. \tag{8}$$

Solving Eq. (8) with respect to z, it is possible to calculate, if it exists, the propagation distance z_D at which the detachment starts:

$$z_D = \frac{v_G b \pm \left(v_G^2 b^2 - 4\Delta n_0 b^2 C^2 \right)^{1/2}}{2\Delta n_0 C^2}. \tag{9}$$

The two solutions refer to the detach situation (when the negative sign of the root is taken) or to the first cross of the centre of the waveguide in the lock-in situation (when the positive sign of the root is taken). Studying the discriminant of Eq. (9) we can derive the value of the amplitude C_D that divides the lock-in from the detach C values:

$$C_D = \left(\frac{v_G^2}{4\Delta n_0} \right)^{1/2}. \tag{10}$$

The above equation demonstrates that high inclination angles of the waveguide raise the amplitude values that can be locked into the waveguide while high values of the refractive index reduce the same values. This perfectly agrees with what one could expect.

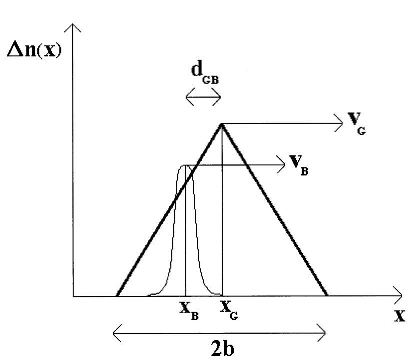

Fig. 2. Relative distance waveguide–soliton at some propagation distance z.

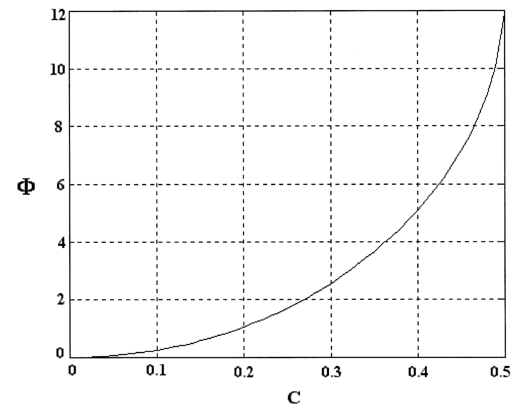

Fig. 3. Detach angle Φ in degrees, equal to arctan(v_D), versus C for $v_G = 0.224$ (waveguide inclination: 12.6°), $\Delta n_0 = 0.05$. The detach value is $C_D = 0.5$

Substituting Eq. (9) into Eq. (6a) it is possible to calculate the detach velocity, that is the inclination according to which a soliton, whose amplitude C is smaller than C_D, leaves the waveguide:

$$v_D = v_B(z_D) = v_G - \left(v_G^2 - 4\Delta n_0 C^2 \right)^{1/2}. \tag{11}$$

A typical detach angle distribution is shown in Fig. 3, where $v_G = 0.224$, corresponding to a waveguide inclination of about 12.6°, and $\Delta n_0 = 0.05$. With these values we have from Eq. (10), $C_D = 0.5$.

The behaviour described above gives to the device a secondary but very important property, that is the capability of working as a demultiplexer controlled by the amplitude of the input beam. For this purpose we develop some other useful concepts.

If we know the point where the beam leaves the waveguide, since the output angle is expressed by Eq. (11), once the length L of the device is given, we can

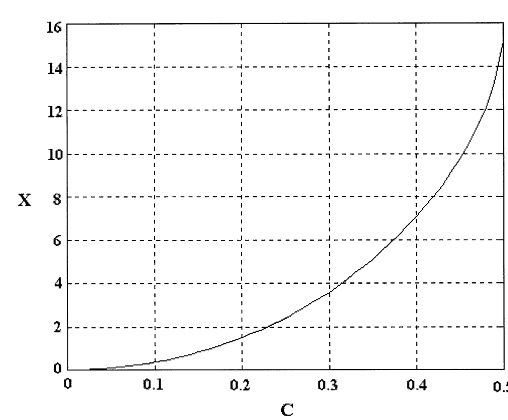

Fig. 4. Position X of the beam at the end of the output of the device as a function of the amplitude C. $v_G = 0.224$, $\Delta n_0 = 0.05$, $L = 100$.

immediately calculate, by means of geometrical considerations, the output position of the beam in the device:

$$x_0 = x_{BD} + (L - z_D) v_D = \frac{b}{4\Delta n_0 C^2}\left(v_G - \sqrt{v_G^2 - 4\Delta n_0 C^2}\right)^2 \times \left(\frac{4L\Delta n_0 C^2}{b\left(v_G - \sqrt{v_G^2 - 4\Delta n_0 C^2}\right)} - 1\right), \quad (12)$$

where x_{BD} is the transverse position of the beam when it leaves the waveguide, that can be calculated substituting Eq. (9) into Eq. (6b):

$$x_{BD} = \frac{b}{4\Delta n_0 C^2}\left(v_G - \sqrt{v_G^2 - 4\Delta n_0 C^2}\right)^2. \quad (13)$$

In Fig. 4 a distribution of the output positions of the device is shown, using Eq. (12).

Different numerical simulations of this structure have been performed, using a FD-BPM method and the results

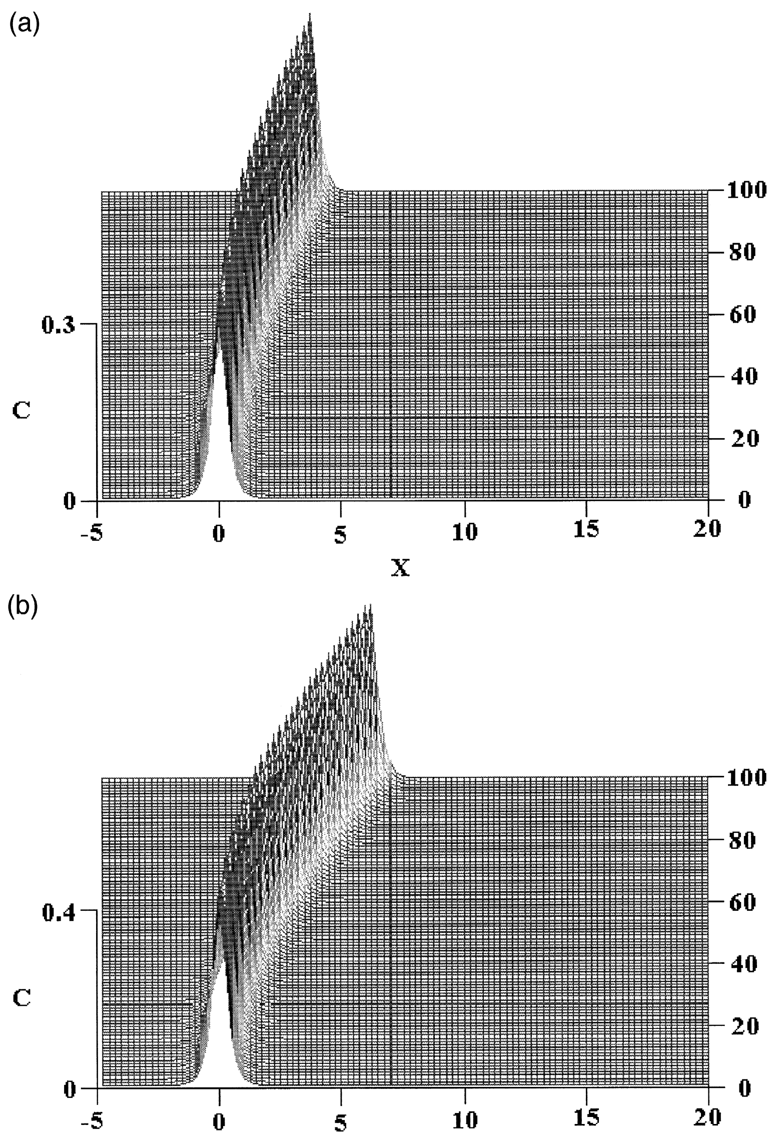

Fig. 5. (a) Numerical simulation for $C = 0.3$ (detach value), $v_G = 0.224$, $\Delta n_0 = 0.05$, $L = 100$. The beam is expelled from the waveguide. (b) Numerical simulation for $C = 0.4$ (detach value), $v_G = 0.224$, $\Delta n_0 = 0.05$, $L = 100$. The beam is expelled from the waveguide. (c) Numerical simulation for $C = 0.5$ (limit value), $v_G = 0.224$, $\Delta n_0 = 0.05$, $L = 100$. The beam propagates along one side of the waveguide. (d) Numerical simulation for $C = 0.6$ (lock-in value), $v_G = 0.224$, $\Delta n_0 = 0.05$, $L = 100$. The beam is locked inside the waveguide.

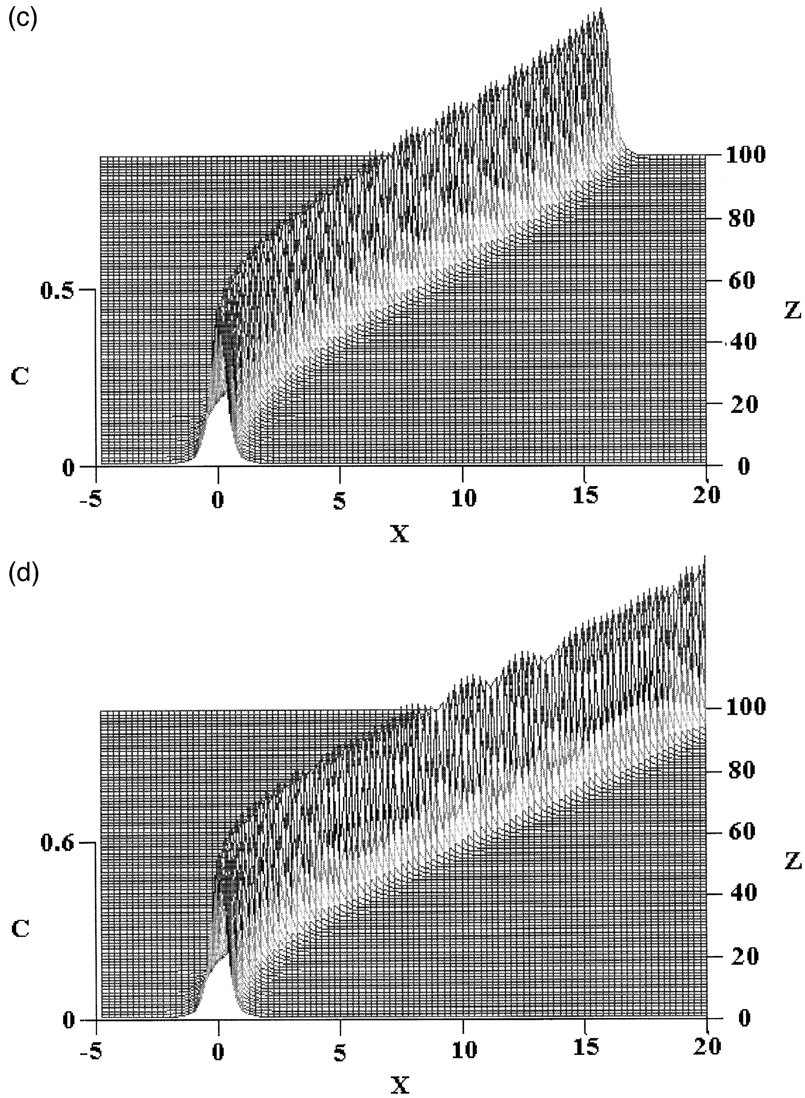

Fig. 5 (continued).

are shown in Fig. 5. The values used are $v_G = 0.224$, $\Delta n_0 = 0.05$, $L = 100$, $b = 5$. With these parameters the detach value was already calculated to be $C_D = 0.5$. It is possible to see that all the beams whose amplitude is below the detach value are expelled from the waveguide according to an angle that can be calculated from Eq. (11). The beam whose amplitude is exactly equal to the detach value propagates on one side of the waveguide, while higher amplitude beams are locked-in. The situation is summarized in Fig. 6 where the position of the waveguide superimposed to the mean position of the beams obtained from numerical simulations is shown. The results perfectly agree with the above developed theory. It is important to note that, due to the particle behaviour of the soliton beam, we have a full power transfer from input to output in both cases of detaching or locking, when other disturbing factors such as scattering, absorption are not considered.

We can see that the lock-in value C_D of the amplitude, expressed by Eq. (10), does not depend on the waveguide length L: this is a direct result of the absence of restrictions about L. In a practical situation it is possible that given a certain waveguide, with some length L, we obtain a lock-in value C_D, whose detachment distance z_D, from Eq. (9), results to be longer than L. In this case it is obvious that, due to the restriction imposed by the waveguide length L, the detach value C_D decreases. In fact, even if the lower values of amplitude tend to be expelled from the waveguide, the expulsion takes place at a distance

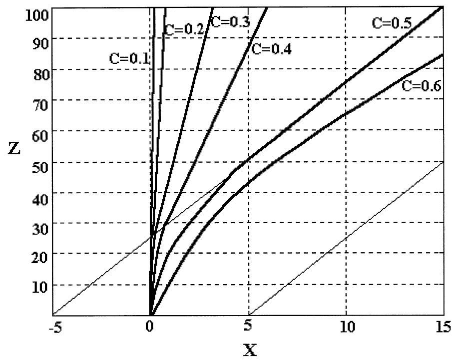

Fig. 6. Upper view of the trajectories of soliton beams obtained from numerical simulations. $v_G = 0.224$ (waveguide inclination: 12.6°), $\Delta n_0 = 0.05$, $L = 100$. The oblique lines represent the sides of the waveguide. The beams characterized by a detach amplitude $(C < 0.5)$ are expelled, the beams characterized by a limit amplitude $(C = 0.5)$ propagate along one side of the waveguide, the beams characterized by a lock-in amplitude $(C > 0.5)$ are locked inside the waveguide.

that is longer than the waveguide and the beam remains trapped. The lower value C_{DL} can be calculated from Eq. (9) setting $z_D = L$ and solving with respect to C:

$$C_{DL} = \frac{b^2}{L}\left(\frac{L v_G - b}{\Delta n_0}\right)^{1/2}. \tag{14}$$

4. A numerical example

We want now to consider a numerical situation.

We consider a beam with full width at half height d_0. The intensity necessary to generate a soliton is [17]:

$$I_s = \frac{2 n_0}{d_0^2 n_2 \beta^2}, \tag{15}$$

where β is the wavevector of the beam, and n_0, n_2 are the linear and nonlinear refractive indices of the medium respectively. The intensity threshold to have a second-order soliton is $I_s^{(2)} = 4 I_s$. This has always to be taken in mind.

It is possible to demonstrate, through some algebra, that in a profile $C\,\mathrm{sech}(Cx)$ the parameters d_0 and C are linked by the relation:

$$\beta d_0 = \frac{2}{C}\log\left(2+\sqrt{3}\right), \tag{16}$$

Substituting Eq. (16) into Eq. (15) and solving with respect to I_s, we obtain:

$$I_s = \frac{1}{\left[\log\left(2+\sqrt{3}\right)\right]^2}\frac{n_0}{2 n_2} C^2, \tag{17}$$

that can be substituted into all the expressions obtained as a function of amplitude C, to convert them into expressions as function of intensity I_s.

Suppose now to have a Kerr material such as a Schott B270 glass [15], whose typical optical parameters at $\lambda_0 = 620$ nm are $n_0 = 1.53$ and $n_2 = 3.4 \times 10^{-20}$ m^2/W. The spot size of the beam is $d_0 = 10$ μm. The physical parameters of the waveguide are: $v_G = 6 \times 10^{-4}$, corresponding to an inclination of about 3.44×10^{-2}°, a waveguide side $b = 10$ μm and a waveguide length $L = 5$ cm. If we choose, for example, $\Delta n_0 = 10^{-4}$, we can calculate, substituting Eq. (10) into Eq. (17), the lock-in intensity that gives $I_D = 1.16 \times 10^{16}$ W/m^2. Using Eq. (15) it is possible to calculate the minimum intensity that is necessary to generate a fundamental soliton, that is $I_s = 3.7 \times 10^{15}$ W/m^2. Since $I_D \cong 3.13 I_s$, if the waveguide would be longer than the lock-in distance, that has not been yet calculated, all solitons whose intensity is below 3.13 times I_s are expelled, while the other ones whose intensity is greater than I_D and less than $4 I_s = 1.27 I_D$ (second order soliton generation) are locked-in. The lock-in distance can be calculated substituting the numerical values into Eq. (9) that gives $z_D = 2.88$ cm, that is a distance shorter than the

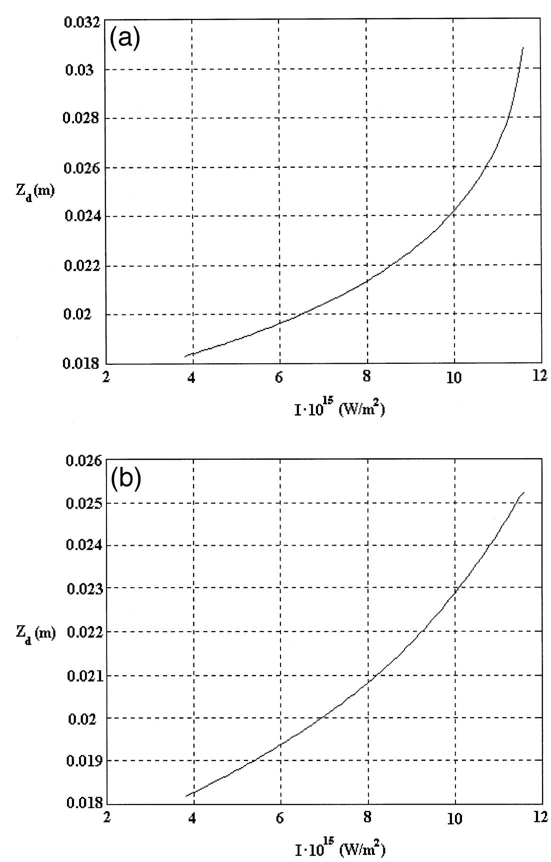

Fig. 7. (a) Detach distance as a function of the input intensity without absorption. (b) Detach distance as a function of the input intensity for an absorption coefficient equal to 30 dB/m.

waveguide length $L = 5$ cm. Using therefore this waveguide the demultiplexing effect is present too. The expelling distance can be rapidly calculated substituting Eq. (17) into Eq. (9) and it is shown in Fig. 7a.

5. Behaviour of the structure in the presence of absorption

We want now to analyze what happens when the material that we are considering is absorbing, that is a realistic situation. Two main events take place in this situation: the detach distance changes and the output intensity is smaller that the input intensity. The variation of the detach distance can immediately be seen from Eq. (9) where it is possible to substitute the amplitude C of the soliton with its intensity I using Eq. (19) that can rapidly be rewritten as

$$I = C^2/C_0, \tag{18}$$

where $C_0 = (2n_2/n_0)\left[\log\left(2+\sqrt{3}\right)\right]^2$. Due to the presence of absorption, the intensity decreases during propagation and the relative detach distance decreases with it. At the detach, the intensity is equal to:

$$I_{\mathrm{ass}} = I\exp(-\alpha z_{\mathrm{D}}), \tag{19}$$

where α is the absorption coefficient of the considered material. Substituting Eq. (18) and Eq. (19) into Eq. (9) we have:

$$z_{\mathrm{D}} = \frac{v_{\mathrm{G}}b - \sqrt{v_{\mathrm{G}}^2 b^2 - 4\Delta n_0 b^2 C_0 I\exp(-\alpha z_{\mathrm{D}})}}{2\Delta n_0 C_0 I\exp(-\alpha z_{\mathrm{D}})}. \tag{20}$$

The above equation gives the detach distance z_{D} as a function of input intensity. Unfortunately it is not possible to solve it in an analytical form due to the presence of the variable z_{D}, in complex form, in both members of the equation. It can only be solved numerically, once all the necessary parameters are given. Fig. 7 shows the detach distances for the numerical situation already considered without absorption and in the presence of an absorption coefficient α equal to 30 dB/m. It can be seen that, with absorption, the curve is scaled and the detach distance, at the same intensity level, decreases. It must be noted that, due to the presence of absorption, it is possible to operate at higher intensity levels because the extra energy is absorbed to reach a longer distance. Anyway, in the figures, we consider the same intensity to compare only the variation of the detach distances.

We want now to consider the variation of input intensity in the absorbing medium. It is obvious that the more the beam propagates inside the material, the more its intensity decreases. Therefore we expect a higher intensity decrease for more intense beams, that are the beams that propagate longer inside the structure. In Fig. 8 the ratio between the output intensity with absorption and the output intensity without absorption is shown versus the input intensity for different values of the absorption coefficient of the medium. It is possible to see that the obtained results agrees with our predictions.

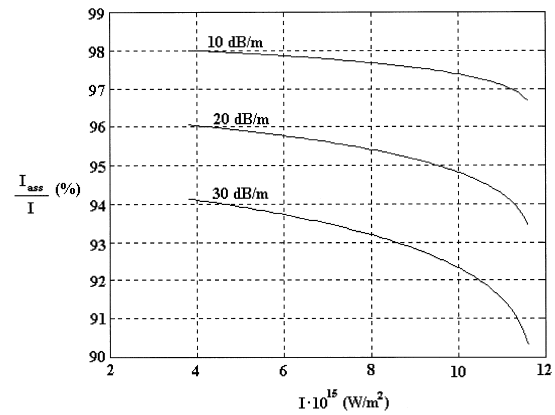

Fig. 8. Ratio between the output intensity with absorption and the output intensity without absorption (percentage) versus input intensity for different values of the absorption coefficient of the medium.

6. Response of the structure to temporal pulses

The above results do not take into account the temporal dimension, that is they consider a constant intensity beam. In a real case we deal with pulses whose intensity varies, growing from zero to a maximum value, to decrease again to zero. This allows us to deduce immediately that, only the temporal part of the pulse, whose intensity is greater than the trapping value depending on the optical parameters of the waveguide, is locked into the structure, while the other parts are expelled.

Let us consider a typical Gaussian pulse, whose amplitude can be written, considering only the temporal part as:

$$A(t) = A_0\exp(-t^2/\tau), \tag{21}$$

where τ is a parameter responsible for the beam width.

Consider a certain waveguide whose lock-in amplitude C_{D} has been calculated. If $A_0 = C_{\mathrm{D}}$, it is evident that only a very narrow part of the central section is trapped inside the waveguide, while most of the pulse is thrown away. The question we want to solve now is how greater must be A_0 with respect to C_{D} ($A_0 = NC_{\mathrm{D}}$) to trap a desired portion of the pulse that is Δt wide. This is equal to say that, considering a portion of the pulse Δt, we want to find Δt as a function of N when:

$$A_0\exp\left(-\frac{\Delta t^2}{\tau}\right) = NC_{\mathrm{D}}\exp\left(-\frac{\Delta t^2}{\tau}\right) = C_{\mathrm{D}}, \tag{22}$$

where N is a real positive number greater than 1.

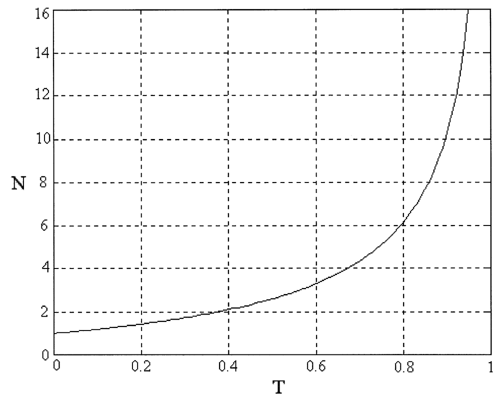

Fig. 9. Magnifying factor N as a function of the trapping percentage T of the beam.

Solving Eq. (22) with respect to Δt we have

$$\Delta t = \sqrt{\tau \log(N)}\,. \tag{23}$$

This is not a very useful result because Δt, as it stands, does not express which percentage of the pulse is trapped. For this purpose we introduce a parameter T, whose values vary in the interval 0–1, that indicates the trapped percentage of the pulse as a function of N, defined as

$$T = \frac{\int_{-\Delta t/2}^{+\Delta t/2} A_0 \exp(-t^2/\tau)\,\mathrm{d}t}{\int_{-\infty}^{+\infty} A_0 \exp(-t^2/\tau)\,\mathrm{d}t} = \mathrm{erf}\left(\frac{\sqrt{\log(N)}}{2}\right). \tag{24}$$

Eq. (24) can be solved with respect to N, to obtain the over-amplitude an input pulse must have with respect to the minimum value C_D, to trap a desired pulse percentage T:

$$N = \exp\left[(2\,\mathrm{erfinv}(T))^2\right]. \tag{25}$$

Eq. (25) is graphically shown in Fig. 9.

If we are interested in the intensity, we have to consider N^2 instead of N.

From Eq. (25) we can see that it is possible to trap any desired portion of the pulse, provided that we use the proper amplitude $A_0 = NC_D$.

This is true from the theoretical point of view, without considering the generation of second order soliton. In fact it is well known that, given a certain beam width and a certain medium, if we inject a beam characterized by an intensity that is four times the intensity of the fundamental soliton calculated with Eq. (15), it generates a second order soliton whose typical profile varies periodically during propagation, with a period equal to what is called soliton period. Since the transverse acceleration strictly depends on the beam profile, the generation of a second order soliton would invalidate all the calculations. It is anyway possible to extend the whole theory to this case, but this is not in the scope of this paper.

Since the intensity must be less than four times the fundamental intensity, the amplitude must be less than two times the fundamental amplitude, and therefore it is immediate to calculate from Eq. (25), or equivalently from Fig. 9, that the maximum trapping percentage T of the pulse is approximately equal to 0.4.

Lower values of amplitude give rise to lower trapping percentage, while higher ones generate second order soliton whose propagation behaviour is quite different with respect to the propagation of the fundamental soliton studied here. The propagation of a second order soliton is outside the scope of this paper.

7. Conclusions

The studied behaviour of a soliton beam in a trapezoidal shaped waveguide allows one to realize a high pass filter where the filtered variable is represented by the intensity of the input beam. This device would be able to work also as a demultiplexer. The cut-intensity is easily controllable through the width and the refractive index of the waveguide.

The transferring ratio between the input and the output in pulsed operation can be accurately controlled by properly choosing the intensity of the input beam.

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