A power Allocation Algorithm for Throughput Maximization in Mobile Networks

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Abstract

This paper focuses on the competitively optimal power-control and signal-shaping for "ad-hoc" networks composed by Multiple-Antenna noncooperative transmit/receive terminals affected by spatially colored Multi-Access Interference (MAI). The target is the competitive maximization of the information throughput (measured in bits/slot) sustained by each link active over the network. For this purpose, the MAI-impaired network is modeled as a noncooperative strategic game, and sufficient conditions for the existence and uniqueness of the Nash Equilibrium are provided. Specifically, the main contribution of this paper may be so summarized. First, we develop fully distributed and scalable power-control and signal-shaping algorithms allowing the implementation of asynchronous Space-Division Multiple Access Strategies (SD-MACSs) able to guarantee the competitive maximization of the users' throughput under both Best Effort and Contracted QoS access policies. Second, we give evidence that the developed SDMACSs outperform (in terms of aggregate throughput) the conventional centralized ones (as TDMA/FDMA/CDMA), specially in operating scenarios affected by strong MAI. Third, we study the convergence property of the presented SDMACSs and show that, when the throughput set requested by the users is no achievable by the network, then the developed SDMACSs are able to move the working point of the system to the nearest one sustainable by the network. Fourth, by exploiting the distributed feature of the presented SDMACSs, we propose two Connection Admission Procedures (CAPs) able to optimize (in a competitive sense) the tradeoff between aggregate networking throughput and connection requirements advanced by the users.

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1 System Modeling

The model here considered supposes that $n^\star$ transmitters and $n^\star$ receivers are present in the network. Each transmitter is equipped with t transmit antennas while the receiver is equipped with r receive ones. The channel is assumed quasi static and the fading model refers to Rayleigh flat fading one. Furthermore the first field of the packet is employed for interference estimation [1] while the second field contains sequences known to the receiver in order to estimate the channel coefficients h_{ji} modelling the path gains from antenna i to antenna j. The first field has T_L length (in multiple of signaling period), the second one T_{TR} while the payload (third field) presents T_{pay} length.

2 Conveyed Information Throughput in the presence of channel estimation errors and spatially colored MAI

The block-fading model introduced for the MIMO channel guarantees that this last is information stable [8], so that the corresponding Shannon' capacity C (nats/slot) fixes the maximum information throughput conveyable in a reliable way during the payload phase [7]. Following quite standard approaches [7], the Shannon' capacity C of the MIMO channel can be expressed as

$$C = E\{C(\hat{\mathbf{H}})\} \equiv \int C(\hat{\mathbf{H}})p(\hat{\mathbf{H}})d\hat{\mathbf{H}}, \ \ \text{(nats/slot)}, \qquad (1)$$

where $p(\hat{\mathbf{H}}) = \left(\frac{1}{\pi(1-\sigma_\varepsilon^2)}\right)^{rt} \exp\left\{ -\frac{1}{(1-\sigma_\varepsilon^2)} Tra[\hat{\mathbf{H}}^\dagger \hat{\mathbf{H}}]\right\}$ is the Gaussian probability density function (pdf) of the channel estimates $\hat{\mathbf{H}}$ [11] of the (t x r) complex matrix $\mathbf{H}$ composed by the path gains h_{ji}, σ_ε^2 is the average squared estimation error of the path gain h_{ji}, and the random variable

$$C(\hat{\mathbf{H}}) \triangleq \sup_{\underline{\vec{\phi}}:\mathrm{E}\{\underline{\vec{\phi}}^\dagger \underline{\vec{\phi}}\} \le tT_{pay}P} \frac{1}{T_{pay}} I\left(\underline{\vec{\mathbf{y}}}; \underline{\vec{\phi}} | \hat{\mathbf{H}}\right), \ (nats/slot) \qquad (2)$$

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is the Shannon' capacity of the MIMO channel *conditioned* on the realization $\hat{\mathbf{H}}$ of the channel estimates actually available at both transmitter and receiver, where $\underline{\mathbf{y}}(n) \triangleq [y_1(n)...y_r(n)]^T$ and $\underline{\phi}(n) \triangleq [\phi_1(n)...\phi_t(n)]^T$ are the vectors collecting the outputs of the r receive antennas over the n-th payload slot and the corresponding signals radiated by the transmit node, while P is the maximal average power that can be radiated by the transmit antennas over each slot. Finally, $I(.;.|.)$ in (2) denotes the mutual information conveyed by the MIMO channel during the payload phase [7]. Unfortunately, barring the limit cases of Perfect Channel State Information (PCSI) and No CSI [2,10], the pdf of the input signals $\overrightarrow{\underline{\phi}}$ achieving the supremum in (2) is *currently unknown*, even for the simplest case of spatially white MAI [10]. However, it is known that Gaussian distributed input signals achieve the supremum in (2) when the condition of PCSI is approached [2], and *also* for imperfect channel estimates when the length T_{pay} of the payload phase (largely) exceeds the number t of transmit antennas (see [10] about this asymptotic result). Therefore, motived by the above considerations, in the following we focus on the evaluation of (2) for *Gaussian distributed* input signals. In this case the T_{pay} components $\{\underline{\phi}(n) \in \mathbb{C}^t, T_L + T_{tr} + 1 \leq n \leq T\}$ of the overall signal vector $\overrightarrow{\underline{\phi}}$ are uncorrelated zero-mean proper complex Gaussian vectors with spatial covariance matrix of the t-dimensional signal vector radiated during each time slot given by $\mathbf{R}_{\phi} \triangleq \mathrm{E}\{\underline{\phi}(n)\underline{\phi}(n)^{\dagger}\}$. The corresponding information throughput

$$\mathbb{T}_G(\hat{\mathbf{H}}) \triangleq \frac{1}{T_{pay}} \sup_{Tra[\mathbf{R}_{\phi}] \leq Pt} I\left(\overrightarrow{\underline{\mathbf{y}}}; \overrightarrow{\underline{\phi}} | \hat{\mathbf{H}}\right), \ (nats/slot) \tag{3}$$

conveyed by the MIMO channel for Gaussian input signals generally falls below the Shannon' Capacity $C(\hat{\mathbf{H}})$ in (2), so that we have $\mathbb{T}_G(\hat{\mathbf{H}}) \leq C(\hat{\mathbf{H}})$. However, the above inequality is satisfied as equality when *at least one* of the above cited two operating conditions (e.g., PCSI or large T_{pay}) is met. Therefore, passing to deal with the evaluation of $\mathbb{T}_G(\hat{\mathbf{H}})$ in (3), we remark that, in general, the conditional mutual information $I\left(\overrightarrow{\underline{\mathbf{y}}}; \overrightarrow{\underline{\phi}} | \hat{\mathbf{H}}\right)$ in (2) *resists* closed-form computation [10]. However, in [11] it is presented the following result.

Proposition 1. Let us assume assigned the spatial correlation matrix $\mathbf{R}_{\phi}$ of the payload streams radiated by the transmit antennas, and let $\mathbf{K}_d$ the covariance matrix of the overall disturbance estimated during the learning phase. Thus, the resulting conditional mutual information $I\left(\overrightarrow{\underline{\mathbf{y}}}; \overrightarrow{\underline{\phi}} | \hat{\mathbf{H}}\right)$ in (2) supported by the MIMO channel for Gaussian input signals admits the following closed-form expression (see [11]):

$$\begin{aligned} I\left(\overrightarrow{\underline{\mathbf{y}}}; \overrightarrow{\underline{\phi}} | \hat{\mathbf{H}}\right) &= \\ = T_{pay} \lg\det &\left(\mathbf{I}_r + \tfrac{1}{t}\mathbf{K}_d^{-1/2}\hat{\mathbf{H}}^T\mathbf{R}_{\phi}\hat{\mathbf{H}}^*\mathbf{K}_d^{-1/2} + \sigma_{\varepsilon}^2 P\mathbf{K}_d^{-1}\right) \\ &- \lg\det\left(\mathbf{I}_{rt} + \tfrac{\sigma_{\varepsilon}^2 T_{pay}}{t}(\mathbf{K}_d^{-1})^* \otimes \mathbf{R}_{\phi}\right), \end{aligned} \tag{4}$$

when *at least one* of the following conditions is met:

$$\text{both } T_{pay} \text{ and } t \text{ are large;} \tag{5}$$

$$\hat{\mathbf{H}} \text{ approaches } \mathbf{H}; \tag{6}$$

$$\text{all SINRs } \gamma_j,\ 1 \leq j \leq r, \text{ in (14) vanish.}\diamond \tag{7}$$

A detailed proof of the above Proposition may be found in [14] and, for sake of brevity, will be not duplicated here.

3 Optimized Power-Allocation and Signal-Shaping in the presence of colored MAI and Channel Estimation errors

Therefore, according to (3), we must proceed to carry out the power-constrained maximization of the conditional throughput (4). For this purpose, let us indicate as

$$\mathbf{K}_d = \mathbf{U}_d\mathbf{\Lambda}_d\mathbf{U}_d^{\dagger}, \tag{8}$$

the Singular Value Decomposition (SVD) of the MAI spatial covariance matrix $\mathbf{K}_d$, where

$$\mathbf{\Lambda}_d \triangleq diag\{\mu_1, ..., \mu_r\}, \tag{9}$$

is the corresponding $(r \times r)$ diagonal matrix of the *magnitude-ordered* singular values of $\mathbf{K}_d$. Thus, after introducing the $(t \times r)$ matrix

$$\mathbf{A} \triangleq \hat{\mathbf{H}}^*\mathbf{K}_d^{-1/2}\mathbf{U}_d, \tag{10}$$

accounting for the *combined* effects of the imperfect channel estimate $\hat{\mathbf{H}}$ and spatial MAI $\mathbf{K}_d$, let us denote as

$$\mathbf{A} = \mathbf{U}_A\mathbf{D}_A\mathbf{V}_A^{\dagger}, \tag{11}$$

the corresponding SVD, where $\mathbf{U}_A$ and $\mathbf{V}_A$ are unitary matrices, while

$$\mathbf{D}_A \triangleq diag\{k_1, ..., k_s, \underline{\mathbf{0}}_{t-s}\}, \tag{12}$$

is the $(t \times r)$ diagonal matrix collecting the $s \triangleq \min\{r, t\}$ *magnitude-ordered* singular-values $k_1 \geq k_2 \geq ... \geq k_s > 0$ of the matrix $\mathbf{A}$. Finally, for future convenience, let us also introduce the following dummy positions:

$$\alpha_m \triangleq \frac{\mu_m k_m^2}{t(\mu_m + P\sigma_{\varepsilon}^2)},\ 1 \leq m \leq s;\ \beta_l \triangleq \frac{\sigma_{\varepsilon}^2 T_{pay}}{t\mu_l},\ 1 \leq l \leq r. \tag{13}$$

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Thus, it can be proven [11,14] that the application of the Kuhn-Tucker conditions [7] allows us to evaluate the optimized transmit powers $\{P^\star(m),\ 1 \le m \le t\}$ achieving the constrained supremum in (3) as detailed by the following *Proposition 2*.

Proposition 2. Let us assume that at least one of the operating conditions listed in (5), (6), (7) is fulfilled. Thus, for $m = s+1, ..., t$, the powers $\{P^\star(m)\}$ achieving the supremum in (3) *vanish*, while for $m = 1, ..., s$ they must be computed according to the following two relationships:

$$P^\star(m) = 0, \text{ when } k_m^2 \le \Big(1+\frac{\sigma_\varepsilon^2 P}{\mu_m}\Big)\Big(\frac{t}{\rho}+\sigma_\varepsilon^2 Tra[\mathbf{K}_d^{-1}]\Big); \quad (14)$$

$$P^\star(m) = \tfrac{1}{2\beta_{min}}\Big\{\beta_{min}L - 1 + \sqrt{\{\beta_{min}L\}^2 + 4\beta_{min}\Big(\rho - \tfrac{1}{\alpha_m} - \tfrac{r\rho\beta_{min}}{\alpha_m T_{pay}}\Big)}\Big\},$$

$$\text{when } k_m^2 > \Big(1+\frac{\sigma_\varepsilon^2 P}{\mu_m}\Big)\Big(\frac{t}{\rho}+\sigma_\varepsilon^2 Tra[\mathbf{K}_d^{-1}]\Big), \quad (15)$$

where $\beta_{min} \triangleq \min\{\beta_l, l = 1, .., r\}$ and $L \triangleq \Big(1-\frac{r}{T_{pay}}\Big)\rho - \frac{1}{\alpha_m}$. Furthermore, the nonnegative scalar parameter ρ in (14), (15) is set so to satisfy the following power constraint:

$$\sum_{m\in\Im(\rho)} P^\star(m) \le Pt, \quad (16)$$

where

$$\Im(\rho) \triangleq \{m = 1, ..., s : k_m^2 > \Big(1+\frac{\sigma_\varepsilon^2 P}{\mu_m}\Big)\Big(\frac{t}{\rho}+\sigma_\varepsilon^2 Tra[\mathbf{K}_d^{-1}]\Big)\}, \quad (17)$$

is the (ρ-depending) set of m-indexes fulfilling the inequality in (15). Finally, the corresponding optimized spatial correlation matrix $\mathbf{R}_{\boldsymbol{\phi}}(opt)$ for the radiated signals is aligned along the right-eigenvectors of the matrix **A** in (10) as in

$$\mathbf{R}_{\boldsymbol{\phi}}(opt) = \mathbf{U}_A diag\{P^\star(1), ...P^\star(s), \underline{\mathbf{0}}_{t-s}\}\mathbf{U}_A^\dagger, \quad (18)$$

so that the resulting maximized information throughput in (3) admits the following (simple) closed-form expression (see [11,14]):

$$\mathbb{T}_G(\hat{\mathbf{H}}) = \sum_{m=1}^{r} \lg\Big(1+\tfrac{\sigma_\varepsilon^2 P}{\mu_m}\Big) + \sum_{m=1}^{s}\Big[\lg(1+\alpha_m P^\star(m)) - \tfrac{1}{T_{pay}}\sum_{l=1}^{r}\lg\Big(1+\beta_l P^\star(m)\Big)\Big] \quad (nats/slot).\diamond \quad (19)$$

4 Some Games Theory Concepts

In order to model the dynamic behavior of the "ad-hoc" network composed by multiple mutually interfering no cooperating pairs of transmit/receive nodes, we resort to the formal tool of the Game Theory [5]. We recall that a noncooperative and strategic game $G \triangleq \langle \mathbb{N}, \mathbb{A}, \{u_g\}\rangle$ has three components [5,13]: a finite set $\mathbb{N} \triangleq \{1, 2, \ldots, n^*\}$ of players, a set $A_g,\ g \in \mathbb{N}$ of possible actions for each player and a set of utility functions. Specifically, after denoting as $\mathbb{A} \triangleq A_1 \times A_2 \times \ldots \times A_{n^*}$ the space of action profiles [13], let us indicate as $u_g : \mathbb{A} \to \mathbb{R}$ the g-th player's utility function. Thus, after indicating by $\underline{\mathbf{a}} \in \mathbb{A}$ an action profile, by $a_g \in A_g$ the players g's action in $\underline{\mathbf{a}}$ and by $\underline{\mathbf{a}}_{-g}$ the actions in $\underline{\mathbf{a}}$ of the other $(n^* - 1)$ players, we can say that $u_g(\underline{\mathbf{a}}) \equiv u_g(a_g, \underline{\mathbf{a}}_{-g})$ maps[1] each action profile $\underline{\mathbf{a}}$ into a real number [13]. In particular, in a strategic noncooperative game each player chooses a suitable action $a_g^\bullet$ from his action set A_g so to *maximize* its utility function, according to the following game rule [13]:

$$a_g^\bullet \equiv max_{a_g\in A_g} u_g(a_g, \underline{\mathbf{a}}_{-g}). \quad (20)$$

Therefore, since there is *no cooperation* among the players, it is important to ensure the dynamic stability of the overall game. A concept which relates to this issue is the so-called *Nash Equilibrium* (NE). Simply stated, a Nash Equilibrium is an action profile $\underline{\mathbf{a}}^\star$ at which no player may gain by unilaterally deviating [5,13]. So, a NE is a *stable operating point* of the Game, because no player has any profit to change his strategy [4,5]. More formally, a NE is an action profile $\underline{\mathbf{a}}^\star$ such that for all $a_g \in A_g$ the following inequality is satisfied [5, 13]:

$$u_g(a_g^\star, \underline{\mathbf{a}}_{-g}^\star) \ge u_g(a_g, \underline{\mathbf{a}}_{-g}^\star), \forall g \in \mathbb{N}, \forall a_g \in A_g. \quad (21)$$

5 The Spatial Power-Allocation Multi-Antenna (SPAM) Game for ad-hoc networks

Let us focus now on the "ad-hoc" network composed by n^* mutually interfering transmit/receive Multi-Antenna units. The ultimate task of the g-th transmit/receive pair is to maximize the information throughput $\mathbb{T}_G(g), g = 1, .., n^*$, sustained by the corresponding link $T_{xg} \to R_{xg}$ via suitable power-allocation and shaping of the signals radiated by T_{xg}. Since the signals radiated by g-th transmitter induces MAI over all other receivers $\{R_{xi}, i \ne g\}$ and the "ad-hoc" nature of the network *does not allow* transmitters to exchange information (e.g., the transmitters do not cooperate), we may model the interaction between transmit/receive pairs active over the network as a noncooperative strategic game [1,4,5]. Specifically, in the considered "ad-hoc" networking scenario the players' set $\mathbb{N}$ is composed by the n^* pairs of trasmitters/receivers, while the set

[1]The notation $u_g(a_g, \underline{\mathbf{a}}_{-g})$ emphasizes that the g-th player controls only own action a_g, but his achieved utility depends also on the actions $\underline{\mathbf{a}}_{-g}$ taken by all other players [5,13].

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of actions A_g available to the the g-th player is the set of all the covariance matrices $\{\mathbf{R}_{\boldsymbol{\phi}}^{(g)}\}$ meeting the usual power constraint over the average power transmit for slot, so we can pose

$$A_g \equiv \{\mathbf{R}_{\boldsymbol{\phi}}^{(g)} : 0 \leq Tra[\mathbf{R}_{\boldsymbol{\phi}}^{(g)}] \leq t_g P_g\},\ g = 1, ..., n^*. \tag{22}$$

This means that the generic action a_g of T_{xg} consists in the transmission of a Gaussian distributed payload sequence with covariance matrix $\mathbf{R}_{\boldsymbol{\phi}}^{(g)}$. Furthermore, the utility function $u_g(.)$ for the g-th transmit/receive pair is the conditional throughput conveyed by the g-th link, so that we can write (see eq.(4))

$$u_g(\underline{\mathbf{a}}) \triangleq u_g(\mathbf{R}_{\boldsymbol{\phi}}^{(1)}, ..., \mathbf{R}_{\boldsymbol{\phi}}^{(g)}, ..., \mathbf{R}_{\boldsymbol{\phi}}^{(n^*)}) \equiv \frac{1}{T_{pay}} I\Big(\overrightarrow{\underline{\mathbf{y}}}^{(g)}; \overrightarrow{\underline{\boldsymbol{\phi}}}^{(g)} | \hat{\mathbf{H}}_g\Big)$$

$$\equiv \lg\det\Big(\mathbf{I}_{r_g} + \frac{1}{t_f}(\mathbf{K}_d^{(g)})^{-1/2}\hat{\mathbf{H}}_g^T \mathbf{R}_{\boldsymbol{\phi}}^{(g)} \hat{\mathbf{H}}_g^* (\mathbf{K}_d^{(g)})^{-1/2} +$$

$$\sigma_\varepsilon^2(g) P^{(g)} (\mathbf{K}_d^{(g)})^{-1}\Big)$$

$$-\frac{1}{T_{pay}} \lg\det\Big(\mathbf{I}_{r_g t_g} + \frac{\sigma_\varepsilon^2(g) T_{pay}}{t_g}((\mathbf{K}_d^{(g)})^{-1})^* \otimes \mathbf{R}_{\boldsymbol{\phi}}^{(g)}\Big), \tag{23}$$

where the g-th MAI covariance matrix $\mathbf{K}_d^{(g)}$ depends on the spatial covariance matrices $\{\mathbf{R}_{\boldsymbol{\phi}}^{(i)}, i \neq g\}$ of the signals radiated by the interfering transmitters [16]. About the rule of the game, each player (e.g., transmitter T_{xg}) chooses the action $\mathbf{R}_{\boldsymbol{\phi}}^{(g)\bullet}$ maximizing the throughput (23) conveyed by own link, so we can write (see (20)):

$$\mathbf{R}_{\boldsymbol{\phi}}^{(g)\bullet} \equiv \arg \max_{\mathbf{R}_{\boldsymbol{\phi}}^{(g)} \in A_g} \Big\{\frac{1}{T_{pay}} I\Big(\overrightarrow{\underline{\mathbf{y}}}^{(g)}; \overrightarrow{\underline{\boldsymbol{\phi}}}^{(g)} | \hat{\mathbf{H}}_g\Big)\Big\}, g = 1, ...n^*. \tag{24}$$

5.1 Competitively Optimal Power-Allocation and Signal-Shaping Algorithms under the Best Effort and Contracted-QoS Policies

In this sub-Section we present the algorithms for the optimized power-allocation and signal-shaping under contracted-QoS and Best Effort policies for the considered networking scenario. Before proceeding, some remarks about the considered QoS policies are in order. We consider the QoS from an information throughput point of view. Specifically, in "ad-hoc" networks with no centralized controllers it is not possible to assure to any user a guaranteed QoS. Thus, in place of guaranteed user's QoS, it is more reasonable, indeed, to resort to the concept of contracted QoS defined according to predefined multiple QoS classes. Since these throughput classes are set according to the multiple QoS requirements that the MAC layer requests from the Physical layer, the algorithm of Table I may be considered an instance of resource allocation algorithm working according to the cross-layer principle. Specifically, the algorithm we present attempts to achieve the target throughput classes dictated by the MAC layer and, if these classes are not achievable due to the MAI, the algorithm attempts to achieve the next lower QoS classes by decreasing the throughput requested by the users. From this point of view, the Best Effort strategy is a *particular case* of the contracted QoS one, where the number of QoS classes approaches infinity.

The algorithm for achieving the maximal throughput over the g-th link under the above introduced Contracted QoS policy is reported in Table I. It must be run by each transmit/receive pair active over the network. In particular, in Table I the Steps from 0 to 11 are set-up procedures and eigen/singular values computations, while $TR_{TH}^{(z)}$ (nats/slot) at the Step 0 is the target throughput defining the z-th QoS class. Step 12 verifies that the Game is playable (e.g., the Nash Equilibrium exists), while Steps 13 and 14 set up the ρ parameter, $\Im(\rho)$ and the step size Δ requested to carry out the power-allocation procedure. The condition at Step 15 assures that the transmit meets the constraint over the power and the Steps from 16 to 18 perform the competitively optimal power-allocation and spatial signal-shaping for the link $T_{xg} \rightarrow R_{xg}$.

In the Steps from 18 to 22 the convergence of the algorithm towards the NE is checked, and in the Step 23 the maximized information throughput sustained by g-th link is evaluated. Finally, Step 24 checks if the achieved throughput is compliant with the QoS requirement. If it is compliant, then the game stops. Otherwise, T_{xg} reduces the overall radiated power of an assigned step-size Δ_l and restarts the game. If the obtained throughput is below the requested one $TR_{TH}^{(z)}$, the transmitter T_{xg} restarts the game with a target throughput $TR_{TH}^{(z-1)}$ lower than the original one $TR_{TH}^{(z)}$.

Thus, about the asynchronous and distributed implementation of the SPAM Game, the key questions are:

- Does Nash Equilibrium exist for the SPAM Game?
- Is the Nash Equilibrium unique?
- Does the above iterative algorithm converge toward the Nash Equilibrium?

The following *Proposition 3* presents sufficient conditions for the existence, uniqueness and achievability of the Nash Equilibrium. From a formal point of view, *Proposition 3* represents the main analytical result of this contribution.

Proposition 3 - With reference to the asynchronous and distributed implementation of the SPAM Game, let us assume

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0. Set the target throughput $TR_{TH}^{(z)}$ of the z-th QoS Classes ;
1. Initialize $\mathbf{R}_{\boldsymbol{\phi}}^{(g)}(new) := \mathbf{R}_{\boldsymbol{\phi}}^{(g)}(old) = [\mathbf{0}_{t_g \times t_g}]$;
2. fl(g)=1;
3. $\mathbb{T}_G(g) = 0$;
4. $\alpha^{(g)} \triangleq (\tilde{P}T_{tr}/r_g)Tra[(\mathbf{K}_d^{(g)})^{-1}]$;
5. $\sigma_\varepsilon^2(g) \triangleq (1 + \alpha^{(g)}/t_g)^{-1}$;
6. Compute and sort the r eigenvalues of $\mathbf{K}_d^{(g)}$;
7. Compute the SVD of $\hat{\mathbf{H}}_g^* \mathbf{K}_d^{(g)-1} \hat{\mathbf{H}}_g^T$;
8. Sort the $s \triangleq min(r,t)$ eigenvalues $\{k_1^{(g)2}, ..., k_s^{(g)2}\}$ of $\mathbf{K}_d^{(g)}$;
9. $\alpha_m^{(g)} \triangleq \mu_m^{(g)} k_m^{(g)2}/t_g(\mu_m^{(g)} + P^{(g)}\sigma_\varepsilon^2(g))$;
10. $\beta_l^{(g)} \triangleq \sigma_\varepsilon^2(g) T_{pay}/\mu_l^{(g)} t_g$;
11. $\mu_{min}^{(g)} \triangleq \min_{1 \le l \le r}\{\mu_m^{(g)}\}$, $\beta_{max}^{(g)} \triangleq \frac{\sigma_\varepsilon^2(g) T_{pay}}{\mu_{min}^{(g)} t_g}$;
12. if $k_m^{(g)2} \ge (\mu_m^{(g)} + P^{(g)}\sigma_\varepsilon^2(g)) \frac{\sigma_\varepsilon^2(g)\sqrt{rT_{pay}}}{\mu_{min}^{(g)}\mu_m^{(g)}}$ for all m
    and fl(g)=1
{
13. Set $\rho^{(g)} := 0$ and $\Im(\rho^{(g)}) := \emptyset$;
14. Set the step size Δ;
15. While $\left(\sum_{m \in \Im(\rho^{(g)})} P^{\star(g)}(m) < Pt_g\right)$ do
{
16. Update $\rho^{(g)} = \rho^{(g)} + \Delta$;
17. Update the set $\Im(\rho^{(g)})$ via eq. (36);
18. Compute the powers and the covariance matrix
    via eq.(33), (34), (37);
}
19. Set $\mathbf{\Psi}^{(g)} := \mathbf{R}_{\boldsymbol{\phi}}^{(g)}(new) - \mathbf{R}_{\boldsymbol{\phi}}^{(g)}(old)$;
20. If $(||\mathbf{\Psi}||_E^2 \le 0.05||\mathbf{R}_{\boldsymbol{\phi}^{(g)}}^{(g)}(old)||_E^2)$
21. then fl(g)=0, else fl(g)=1;
22. $\mathbf{R}_{\boldsymbol{\phi}}^{(g)}(old) := \mathbf{R}_{\boldsymbol{\phi}}^{(g)}(new)$
}
23. Evaluate $\mathbb{T}_G(g)$ via (38) for the g-th link;
24. if $\mathbb{T}_G(g) = TR_{TH}^{(z)}$ stop; else
25. if $\mathbb{T}_G(g) > TR_{TH}^{(z)}$ reduce the radiated power $P^{(g)}$
    and go to Step 1, else
26. if $\mathbb{T}_G(g) < TR_{TH}^{(z)}$ lower the target class to z-1
    and go to Step 1;
}

Table 1. A pseudo-code for the implementation of the power-allocation and signal-shaping algorithm for the g-th transmitter/receiver pair under the "contracted QoS" policy.

that the following three conditions are met:

$$k_m^{(g)2} > \left(1 + \frac{\sigma_\varepsilon^2(g)P^{(g)}}{\mu_m^{(g)}}\right)\left(\frac{t}{\rho^{(g)}} + \sigma_\varepsilon^2(g)Tra[(\mathbf{K}_d^{(g)})^{-1}]\right),$$
$$\text{width } 1 \le m \le min\{r_g, t_g\}, 1 \le g \le n^*; \quad (25)$$

$$r_g \ge t_g,\ 1 \le g \le n^*; \quad (26)$$

$$T_{pay} >> t_g > 1\ and/or\ \sigma_\varepsilon^2(g) \to 0,\ 1 \le g \le n^*. \quad (27)$$

Thus, the Nash Equilibrium of the SPAM Game exists and it is unique. Furthermore, the distributed and asynchronous implementation of the SPAM Game converges to the NE from any starting point.$\diamond$

6 Network Throughput performance and convergence property

To test the effectiveness of the proposed SPAM algorithm, several numerical tests have been carried out. The obtained results about the achieved throughput and the convergence property of the SPAM Game are detailed in the following sub-Sections.

6.1 Conveyed Network Throughput

Fig.1 depicts the basic squared network considered for the tests. It is composed by two transmit/receive pairs equipped with t=4 and r=8 transmit/receive antennas and operating at SNR=10dB with $T_{pay} = 120$. The numerical tests have been carried out under the Best Effort policy. At the beginning (e.g., at iteration 0), only the first transmit/receive pair is assumed to be on (see Fig.1). Thus, by running the SPAM Game we obtain an information throughput around 18 bits/slots for the first pair (see Fig.2), while the throughput sustained by the second link $T_{x2} \to R_{x2}$ is (obviously) zero. Next, the link $T_{x2} \to R_{x2}$ is activated, so the throughput over the $T_{x1} \to R_{x1}$ link decreases (till to 13 bits/slots; see Fig.2), while the throughput of the $T_{x2} \to R_{x2}$ link increases till the same value of 13 bits/slot. This point represents the (first) Nash Equilibrium for the considered squared topology and it has reached after 23 iterations (see Fig.2). Next, the network topology changes and an obstacle is introduced between the the second transmitter and first receiver, so that $\chi^2(1,2) = 0.6$, while $\chi^2(2,1) = 1$, where $\chi(f,g)$ accounts for the shadowing effects [2] possibly present on the link $T_{xf} \to R_{xg}$. As it can be seen by Fig.2, the new Nash Equilibrium (achieved at the 60th iteration) is characterized by *different* values of the achieved throughput over the active links. After, we considered an operating scenario with $\chi^2(1,2) = 0.8$ and $\chi^2(2,1) = 1$. In this case the SPAM Game gives arise to an information throughput over the link $T_{x1} \to R_{x1}$ limited up to 14.3 bits/slot (see Fig.2). Next, we introduced an additional change in the network topology so that both receivers do not suffer from MAI

[2] Without loss of generality, we may assume the parameter $\chi(f,g)$, spanning the interval $[0,1]$. This means that $\chi(f,g) = 1$ gives arise to the worst case when the MAI effects induced by T_{xf} on R_{xg} are maximal, while $\chi(f,g) = 0$ describes the lucky operating condition where no MAI is induced by T_{xf} on R_{xg}.

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(e.g.,$\chi^2(1,2) = \chi^2(2,1) = 0$; see Fig.1). In this operating condition, the sustained links throughput increase so to approach a new NE, where the throughput conveyed by both links equates 19.2 bits/slot (see Fig.2). Finally, we assumed that a third pair of trasmit/receive units switch on, so that the network assumes a hexagonal topology (see Fig.1). The new NE achieved by running the SPAM Game approaches 8.3 bits/slot for all active links (see Fig.2). About the convergence property, an interesting still open question concerns the convergence rate of the SPAM Game towards the NE for increasing values of the number k of performed iterations. By fact, this question is still open and till now the convergence rate seems to resist, indeed, closed-form analytical evaluation. However, the achieved numerical results support the *conjecture* that this convergence rate is (at least) exponential in the number k of the iterations performed by each transmit/receive pair, according to the following (empirical) relationship:

$$\sum_{g=1}^{n^*} ||\mathrm{E}\{\mathbb{T}_G^{(g;k)}\} - \mathrm{E}\{\mathbb{T}_G^{*}{}^{(g)}\}||^2 \leq (n^* - 1) r_{\max} 2^{-k},$$

where $\mathbb{T}_G^{(g;k)}$ in the throughput conveyed by the g-th link at the k-th iteration of the algorithm of Table I, $\mathbb{T}_G^{*}{}^{(k)}$ is the corresponding throughput achieved at the NE, $r_{\max} \triangleq \max_{1 \leq g \leq n^*}\{r_g\}$, and the expectations are over the fading coefficients of the MIMO channels composing the network. Analytical validation of the above relationship is currently under investigation by the authors.

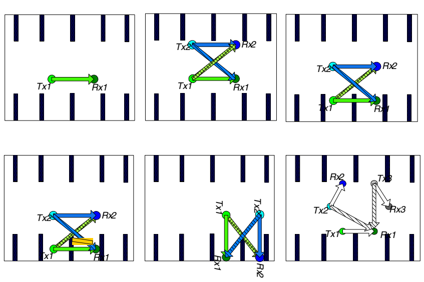

Figure 1. The considered network topology sequence for the numerical tests.

6.2 The Achievable Throughput Regions

The set of simultaneous throughput achieved by the n^* links $T_{xg} \rightarrow R_{xg}, g = 1,..,n^*$ active over the ad-hoc network may be described by resorting to the concept of *achievable throughput region* [2,3]. Roughly speaking, for a given statistical description of the network links and a set

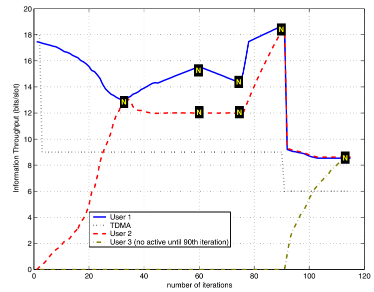

Figure 2. Information Throughput achieved by the SPAM Game and TDMA under the Best Effort policy for the network topology sequence of Fig.1.

of constraints on the network input statistics (power, distribution, etc.), the corresponding *throughput region achievable* by the overall network is the closure of all information throughput n^*-ples $(\mathbb{T}_G(1), ..., \mathbb{T}_G(n^*))$ in (23) that can be *simultaneously sustained* by the communication channels $T_{xg} \rightarrow R_{xg}, g = 1, ..., n^*$, active over the network [9,13]. Barring some partial contributions, till now no closed-form analytical formulas are available for the computation of the achievable throughput region of an interference network [12,18,19]. Thus, in this sub-Section we comment some results we have numerically obtained for a squared network composed by two (e.g.,$n^* = 2$) multi-antenna (e.g., $t_1 = t_2 = r_1 = r_2 = 4$) transmit/receive units. Specifically, Fig.3 reports the achievable throughput regions of the considered squared network for different values of the shadowing factors $\chi^2(1,2) = \chi^2(2,1)$. These regions represent the 2-ples of information throughput $(\mathbb{T}_G(1), \mathbb{T}_G(2))$ that the links active over the considered network may guarantee when the proposed SPAM Game is run. After comparing the throughput regions achieved by the proposed SPAM Game with those of the conventional TDMA orthogonal access method (see the inner square in Fig.3), we may conclude that at $\chi^2(1,2) = \chi^2(2,1) < 0.7$ (e.g.,in the presence of strong MAI) the proposed SPAM Game outperforms the conventional TDMA one in terms of conveyed throughput.

6.3 SPAM Game-vs.-TDMA: a throughput comparison

The above conclusion is also supported by the dotted curve of Fig.2 that reports the throughput achieved by the standard TDMA for the same networking scenarios previously considered in Sect.6.1 and sketched in Fig.1. In fact,

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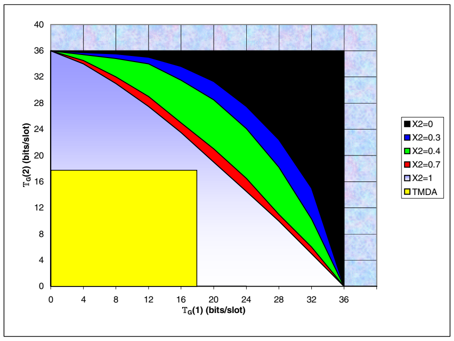

Figure 3. Regions for the information throughput achieved by the proposed SPAM Game and TDMA for a squared network and different values of the shadowing factors $\chi^2(1,2) \equiv \chi^2(2,1) \equiv \chi^2$.

an examination[3] of Fig.2 shows that, although the TDMA is a *centralized* technique assuring *orthogonal* (e.g., collision free) multiple access, nevertheless the corresponding throughput are below than those achieved by running the proposed SPAM Game, specially when the MAI effects are substantial. Overall, the SPAM Game-vs.-TDMA comparison of Fig.2 supports for the superiority of decentralized competitively optimal access strategies over centralized orthogonal ones, at least in networking scenarios where the spatial-dimension of of the system may be efficiently exploited to perform MAI suppression.

6.4 Convergence Property of the SPAM Game towards the nearest allowable operating point

In actual application scenarios, the transmit/receive nodes are not aware in advance about the throughput region of Fig.3 sustainable by the network, neither this region may be analytically evaluated in closed-form. Thus, a key question concerns the convergence of the operating point of SPAM Game when the requested initial throughput $(\mathbb{T}_G^{(0)}(1), \mathbb{T}_G^{(0)}(2))$ fall out of the achievable throughput region of Fig.3. We have observed that, under the BE policy, the operating point of the SPAM Game moves from $(\mathbb{T}_G^{(0)}(1), \mathbb{T}_G^{(0)}(2))$ and converges to the point on the *boundary* of the throughput region at the *minimum Euclidean distance* from the initial $(\mathbb{T}_G^{(0)}(1), \mathbb{T}_G^{(0)}(2))$ point (see the dotted arrow of Fig.4). Likewise, under the Contracted QoS policy, the operating point of the SPAM Game moves from $(\mathbb{T}_G^{(0)}(1), \mathbb{T}_G^{(0)}(2))$ and converges to the point *on the QoS grid* at minimum distance from $(\mathbb{T}_G^{(0)}(1), \mathbb{T}_G^{(0)}(2))$ (see the dashed grid of Fig.4).

[3]Under the above stated assumptions about the considered network in Fig.1, the same throughput values marked by the dotted plot of Fig.2 are also achieved when alternative *orthogonal* centralized access strategies (as, for example, CDMA or FDMA) are implemented.

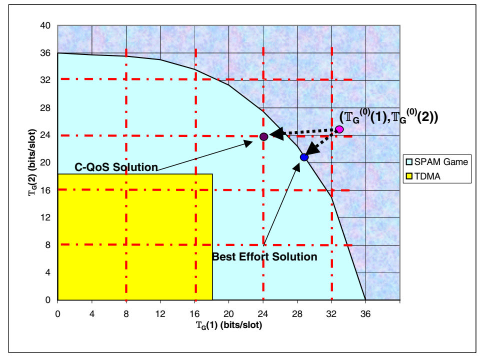

Figure 4. Regions for the information throughput achieved by the proposed SPAM Game and TDMA for a squared network with $\chi^2(1,2) \equiv \chi^2(2,1) \equiv 0.4$.

7 Distributed Connection Admission Procedures (CAPs) and Conclusions

Since in "ad-hoc" networks no centralized controllers are present and, in addition, the number of active nodes may vary in an unpredictable way, an interesting topic concerns the development of *distributed* and *scalable* CAPs balancing QoS users' requirements-vs.-aggregate networking throughput. In this Section, we propose two distributed CAPs for "ad-hoc' networks relying on the above described SPAM Game. The first one (referred to as Hard Connection Admission Procedure (HCAP)) attempts to satisfy the users requiring higher QoS classes, while the goal of the second one (e.g., the Soft Connection Admission Procedure (SCAP)) is to maximize the number of allowed connections. The flow chart of the HCAP is reported in Table 2. To understand it, let us assume that, after the network approached the NE, a new request by an incoming user with QoS class z arises. Thus, the algorithm for the power allocation of Table I starts and the new user evaluates the changes of own interference covariance matrix $\mathbf{K}_d$. Obviously, also all other users already active over the network track the changes of their interference covariance matrices. If the convergence is achieved (e.g., no changes in the MAI covariance matrices of all users are recognized), then a new NE is approached.

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Otherwise, each user waits a *time zT proportional*[4] to its own required QoS class. Afterwards, if changes in the MAI matrices occurred, then the user with a QoS class equal to z waits for a time equal to zT before decreasing his QoS class to $(z-1)$. Afterwards, the power-allocation algorithm restarts once a time (see Table 2).

1. Equilibrium for the network composed by (n^*-1) pairs of transmit/receive nodes;
2. New Request from user n^* with QoS Class equal to $z^{(n^*)}$;
3. Power allocation for g-th user , $g=1,...,n^*$;
4. The user g waits for $z^{(g)}T$ for the network equilibrium, $g=1,...,n^*$;
5. If the network is in equilibrium, go to Step 1;
6. else $z^{(g)} := z^{(g)}-1$, $g=1,...,n^*$;
7. Go to Step 3.

Table 2. Hard Connection Admission Procedure (HCAP).

1. Equilibrium for the network composed by (n^*-1) pairs of transmit/receive nodes;
2. New Request from user n^* with QoS Class equal to $z^{(n^*)}$;
3. Power allocation for the g-th user , $g=1,...,n^*$;
4. The g-th user waits for $(z_{max}-z^{(g)})T$ for the network equilibrium, $g=1,...,n^*$;
5. If the network is in equilibrium, go to Step 1;
6. else $z^{(g)} := z^{(g)}-1$, $g=1,...,n^*$;
7. Go to Step 3.

Table 3. Soft Connection Admission Procedure (SCAP).

In Table 3 the flow-chart of SCAP is reported. The SCAP approach is quite similar to HCAP, barring for the convergence time (that is *inversely proportional* to the required QoS class) and for the reduction of classes. In fact, in this approach the user with *higher class* is the first to reduce its QoS class so to attempt to maximize the overall number of allowed connections. The numerical plots of Figs.5,6 support the actual effectiveness of the proposed CAPs.

Specifically, in Fig.5 we evaluate the number of users that are able to establish a connection as a function of the percentage of QoS users. The SCAP allows connections to a number of users higher than that of HCAP, while an orthogonal access method (e.g., TDMA) allows the connection to all users, regardless of the number of QoS users. The number of users connected with both SCAP and HCAP decreases when the percentage of QoS users increases. In Fig.6 the achieved aggregated network throughput is reported. Although the number of users connected with HCAP approach is the *lowest*, nevertheless the network throughput achieved by this last is the *highest*. The SCAP offers the connection to a number of users *higher* than that of HCAP, but the overall network throughput is *lower*. Fig.6 confirms that, in terms of aggregate network throughput, the TDMA presents the worst performance.
Overall, the final conclusion that arises from the performance tests described in Sects. 6,7 is that the proposed SPAM Game represents a distributed Multi-Antenna access

[4]The calue assumed by the waiting time may be set (possibly in an adaptive way) by the MAC layer on the basis of the maximum delay (e.g., latency) allowed for the (successful) transmission of each MAC PDU.

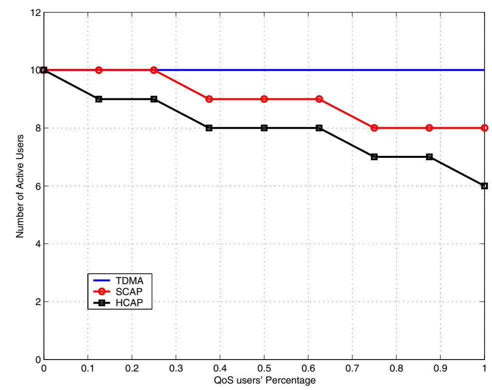

Figure 5. Number of connected users for a random network topology. Performance comparison between TDMA, SCAP and HCAP. The QoS-classes are the same shown in Fig.4.

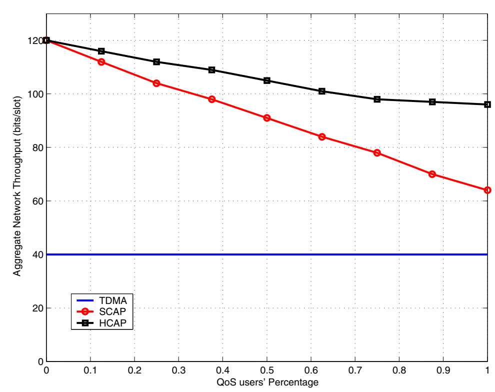

Figure 6. Aggregate Network Throughput for the same scenario of Fig.5.

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strategy able to outperform (both in term of aggregate throughput and number of allowed connections) the conventional centralized ones as TDMA/FDMA/CDMA. This conclusion may be of actual interest for a "plug-and-play" planning of Multi-Antenna "ad-hoc" network architectures. From this point of view, it is likelihood to retain that the results presented in this paper only grasp the tip of the iceberg and much remains to be done. Specifically, the effect of multi-hop routing and relays [24] on the performance of the proposed SPAM Game is a topic currently investigated by the authors.

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